Solution to Test 2
October 14, 2020

This test consists of three parts. Please note that in parts II and III, you can skip one question of those offered.

Part I: Multiple Choice [20 points]
For each question, choose the best answer (2 points each). Your test will have had these questions in a random order.

2. Which of the following were innovations that Bohr applied when he built his model of the hydrogen atom?
   A) When electrons move from one level to another, they emit or absorb one photon
   B) Electrons orbit the nucleus in ellipses, not circles
   C) Electrons can orbit with any angular momentum, not just multiples of h-bar
   D) The positive charge is concentrated in the nucleus, instead of spread throughout the atom
   E) The nucleus contains most of the mass of the atom, instead of the electron

3. If you double the temperature of a black body, by what factor would the amount of power coming from it increase?
   A) Muliply by 1/2
   B) Multiply by 2
   C) Muliply by 4
   D) Multiply by 8
   E) Multiply by 16

4. If everything has wave properties, how come they don't cause diffraction for macroscopic objects like people?
   A) They are significant, that's why we often find ourselves wandering around saying, "now, where was I going?"
   B) The position of a large object can be determined with arbitrary precision, making wave properties negligible
   C) Large objects have wavelengths much larger than anything they interact with, making their wave properties negligible
   D) Their wavelengths are so short that any diffraction is tiny
   E) People are composite objects, and quantum mechanics only applies to elementary particles, not composites.

5. Simplify $e^{i\pi/2}$
   A) -1 B) 1 **C) $\boldsymbol{i}$** D) $-i$ E) None of the above

6. The size of an atom divided by the size of a nucleus is about what number?
   A) 100 B) 1,000 **C) 10,000** D) 100,000 E) 10,000,000

7. Deuterium is a variation on hydrogen where the nucleus contains a proton and a neutron, instead of just a proton. Why are the spectral lines of deuterium slightly different from hydrogen?
   A) The neutron contains a tiny charge, shifting the energies
   B) The neutron pushes the proton away from the center, slightly changing the energies
   C) The nucleus has a different mass, and therefore the nucleus shifts differently as the electron oribts it a little different from hydrogen
   D) The neutron can interact with the electron slightly, shifting the energy
   E) Some of the emission comes from the neutron, causing extra spectral lines to appear

8. What was special about Millkan's oil drop experiment?
   A) It measured the mass of the electron
   B) It could see the effects of the gain or loss of a single unit of charge
   C) It could accuratly measure the strength of electric fields
   D) It demonstrated that most of the mass of an atom was in the nucleus
   E) It could tell that the charges that moved were negative, not positive

9. In order to get interference effects with light, how fast must you put photons through an experiment, such as a two slit experiment?
   A) You must put all the photons through simultaneously
   B) There should be several photons somewhere in the experiment at each moment
   C) There must be enough photons passing through that they can collectively act like a wave, instead of a particle
   D) There must be two photons passing through the slits (one each) at the same time
   E) There is no minimum; you can do it even if there is only one photon at a time in the experiment

10. The Bohr model works pretty well to predict the energies for electrons falling into the innermost levels of many-electron atoms, but you have to use the formula as if the charge of the nucleus were $Z-1$ instead of the actual charge Z. Why?
   A) The uncertainty principle guarantees that some of the charge is not actually in the nucleus
   B) There are other electrons cancelling out or "screening" the nuclear charge a little bit
   C) The innermost electron repels other charges, effectively reducing its charge
   D) The electron is so close to the nucleus that it effectively shares some of its charge with the nucleus, canceling it a little
   E) The electron is partly inside the nucleus, and any charge farther away than it doesn't contribute, by Gauss's Law

11. If the relation between angular frequency and wave number is $\omega = Ak^2 + Bk + C$, then what is the formula for the group velocity v_g?
   A) $Ak + B + \frac{C}{k}$ B) $2Ak + B + \frac{C}{k}$ **C)** $2Ak + B$ D) $2Ak^2 + Bk$ E) $Ak + B$

Part II: Short essay [20 points]

Choose two of the following three questions, and write a short essay (2-3 sentences). You may type both answers into the answer box at the end, or you may upload your answers as an image into the box. Each question is worth 10 points.

12A. What did Max Planck have to assume about the energies in each mode of electromagnetic waves in order to get rid of the ultraviolet catastrophe? You should include at least one equation.

Classically, electromagnetic waves can have any amount of energy between zero and infinity, but Bohr assumed that the energy had to be an integer multiple of hf; that is, $E = nhf$, with n a non-negative integer, where h was a new constant we now call the Planck constant.

12B. An electron in a hydrogen atom would have its lowest energy if it is at rest and at the same location as the nucleus. So is this what happens? Explain.

In quantum mechanics, it is impossible to specify both the position and the momentum. The uncertainty principle says that the product of the uncertainties of these two objects has a lower limit, implying they cannot be both arbitrarily small.

12C. In the Frank-Hertz experiment, electrons are accelerated through a thin gas with gradually increasing voltages (speeds), and the resulting current is measured. What causes there to suddenly be a decrease in the current?

Because atoms have only discreet energy levels, it is impossible to input a small amount of energy into the atom, and hence below a certain threshold, all collisions must be completely elastic, and no energy is lost. When the voltage becomes adequate to excite the atoms, the electrons can collide inelastically, losing energy and this causes a decrease in the current.

Part III: Calculation: [60 points]

Choose three of the following four questions. Each question is worth 20 points. Type only your answers to each part into the essay box provided.

13. Aluminum has a work function of 4.16 eV. Light is shone on the aluminum, and then a voltage is set up that repels the electrons if they don't have enough energy to overcome it. For each of the following wavelengths, find the voltage V_{max} such that the electrons cannot make it through the barrier, or write "impossible" if no electrons can be emitted.

a) 205 nm b) 257 nm c) 327 nm

We use the photoelectric effect equation $eV_{max} = hf - \phi$ together with the wave equation for light $f\lambda = c$ to find

$$eV_{max} = hf - \phi = \frac{hc}{\lambda} - \phi = \frac{1239.8\ \text{eV}\cdot\text{nm}}{\lambda} - 4.16\ \text{eV}$$

We find, therefore, the three voltages from

$$eV_{max} = \frac{1239.8\ \text{eV}\cdot\text{nm}}{205\ \text{nm}} - 4.16\ \text{eV} = 6.05\ \text{eV} - 4.16\ \text{eV} = 1.89\ \text{eV}\,,$$

$$eV_{max} = \frac{1239.8\ \text{eV}\cdot\text{nm}}{257\ \text{nm}} - 4.16\ \text{eV} = 4.82\ \text{eV} - 4.16\ \text{eV} = 0.66\ \text{eV}\,,$$

$$eV_{max} = \frac{1239.8\ \text{eV}\cdot\text{nm}}{327\ \text{nm}} - 4.16\ \text{eV} = 3.79\ \text{eV} - 4.16\ \text{eV} = -0.37\ \text{eV}\,.$$

For the first two, simply cancel the factor of e to get the voltage. For the third one, because the energy of the photon is less than the work function, the electron cannot be freed at all, so the result is impossible. So our final answers are:

a) $V_{max} = 1.89\ \text{V}$

b) $V_{max} = 0.66\ \text{V}$

c) impossible

14. A carbon atom (*Z* = 6) has a single electron in it in the *n* = 5 state. Suddenly, the electron transitions to the *n* = 4 state.

a) What is the energy of the initial and final state in eV?

We use the formula $E_n = -(13.6\text{ eV})Z^2/n^2$, so we have

$$E_5 = -\frac{(13.6\text{ eV})6^2}{5^2} = -19.58\text{ eV},$$
$$E_4 = -\frac{(13.6\text{ eV})6^2}{4^2} = -30.60\text{ eV}.$$

b) To perform this transition, will it emit or absorb a photon? Find the energy of the corresponding photon.

Since the energy is decreasing (becoming more negative), it must be emitting a photon. The energy emitted is the initial energy minus the final energy, so

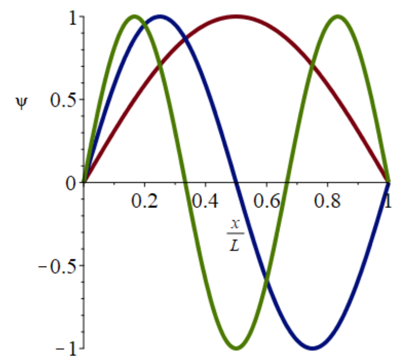
$$E = E_5 - E_4 = -19.58\text{ eV} - (-30.60\text{ eV}) = 11.02\text{ eV}.$$

c) Find the frequency (in Hz = s^{-1}) and wavelength (in nm) of the corresponding photon.

We use the formulas $E = hf$ and $f\lambda = c$ to find the frequency and wavelength, which are

$$f = \frac{E}{h} = \frac{11.02\text{ eV}}{4.1357\times10^{-15}\text{eV}\cdot\text{s}} = 2.664\times10^{15}\text{ s}^{-1} = 2.664\times10^{15}\text{ Hz},$$
$$\lambda = \frac{c}{f} = \frac{hc}{E} = \frac{1239.8\text{ eV}\cdot\text{nm}}{11.02\text{ eV}} = 112.5\text{ nm}.$$

15. In introductory physics, you learned that when a wave is trapped in a region in one dimension of size *L*, the number of half-wavelengths fitting into that region must be an integer, so $\frac{1}{2}n\lambda = L$, where $n = 1, 2, 3, \ldots$. For example, the first three waves are sketched at right.



a) Assuming this relation applies in quantum mechanics, solve for the wavelength *λ* as a function of *n* and then use the de Broglie relation to find the corresponding momentum.

We multiply both sides by $2/n$ to solve for the wavelength, then we use the formula $p\lambda = h$ to find the wavelength:

$$\lambda = \frac{2L}{n},$$
$$p = \frac{h}{\lambda} = \frac{hn}{2L}.$$

b) Find an expression for the energy as a function of *n*, the mass of the particle *m*, and the length *L*.

The energy is given by $E = \frac{1}{2}mv^2$, and writing $v = p/m$ we have

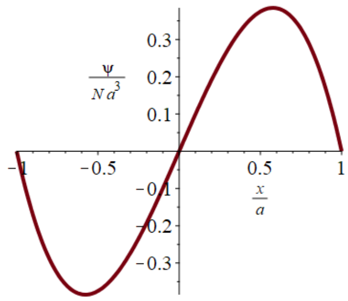
$$E = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{p}{m}\right)^2 = \frac{p^2}{2m} = \frac{1}{2m}\left(\frac{hn}{2L}\right)^2 = \frac{h^2n^2}{8mL^2}.$$

c) For an electron in a box of size 0.570 nm, find the energy in the ground state (*n* = 1) in eV.

Substituting $n = 1$ and including the electron mass, we have

$$E = \frac{h^2}{8mL^2} = \frac{\left(6.6261\times10^{-34}\ \text{J}\cdot\text{s}\right)^2}{8\left(9.1094\times10^{-31}\ \text{kg}\right)\left(0.570\times10^{-9}\ \text{m}\right)^2} = 1.851\times10^{-19}\ \text{J}$$
$$= \frac{1.851\times10^{-19}\ \text{J}}{1.602\times10^{-19}\ \text{J/eV}} = 1.156\ \text{eV}.$$

16. A particle has a wave function in the region $-a < x < a$ given by $\psi(x) = N\left(a^2x - x^3\right)$, where N is a normalization constant, as sketched at right. Some useful integrals are given at the end of this problem.



a) In the region $-a < x < a$, what position(s) is it impossible for the particle to be?

The places it is impossible to be is where the wave function vanishes, so when

$$0 = \psi(x) = N\left(a^2x - x^3\right) = Nx\left(a^2 - x^2\right) = Nx(a - x)(a + x),$$

$$x = 0 \quad \text{or} \quad x = a \quad \text{or} \quad x = -a.$$

Technically, only the first one lies in the specified region.

b) In the region $-a < x < a$, what position(s) is the particle most likely to be?

The most likely place is anywhere where the function is most positive or most negative. To find these points, we take the derivative and set it equal to zero, so we have

$$0 = \frac{d}{dx}\psi(x) = N\frac{d}{dx}\left(a^2x - x^3\right) = N\left(a^2 - 3x^2\right),$$

$$3x^2 = a^2,$$

$$x = \pm\frac{a}{\sqrt{3}}.$$

c) Assuming the wave function vanishes outside this region, what must be the normalization factor N?

The normalization factor is chosen so that the normalization integral yields one; that is

$$1 = \int_{-\infty}^{\infty} |\psi(x)|^2\, dx = \int_{-a}^{a} N^2\left(a^2x - x^3\right)^2 dx = N^2\frac{16}{105}a^7,$$

$$N^2 = \frac{105}{16a^7},$$

$$N = \sqrt{\frac{105}{16a^7}} = \frac{\sqrt{105}}{4}a^{-7/2}.$$

Possibly Helpful Integrals:

$$\int_{-a}^{a}\left(a^2x - x^3\right)dx = \int_{-a}^{a}\left(a^2x - x^3\right)^3 dx = 0, \quad \int_{-a}^{a}\left(a^2x - x^3\right)^2 dx = \frac{16}{105}a^7.$$