December 11, 2019 Name ________________

Solutions to Final Exam

This test consists of five parts. Please note that in parts II through V, you can skip one question.

Part I: Multiple Choice (mixed new and review questions) [50 points] (2 points each)
For each question, choose the best answer

1. According to general relativity, if you go inside the Schwarzschild radius for a black hole, nothing can escape, not even light. But the same formula applies for the metric around the Earth. Why don't we notice this effect?
   A) There is light from the Sun in our area, which makes the Earth not black
   B) The Schwarzschild radius is relevant only for high mass objects, and the Earth is too low mass for there to be such an effect
   C) The Schwarzschild radius is deep within the Earth; if we could somehow reach this, then we would observe this effect
   D) The Schwarzschild metric applies only outside the Earth, and the Schwarzschild radius is inside the Earth, where the formula doesn't apply
   E) It does apply, but because we are inside the Earth's Schwarzschild radius, we can see the light

2. The particle that exchanges the force between the nucleons in a nucleus are called
   A) Neutrons B) Neutrinos C) Photons D) W-bosons **E) Pions**

3. If an electron has $l = 3$, what would be the value of the angular momentum squared $\vec{L}^2$?
   A) $3\hbar^2$ B) $6\hbar^2$ C) $9\hbar^2$ **D)** $12\hbar^2$ E) $18\hbar^2$

4. Which of the following is not a prediction of special relativity
   A) Moving clocks slow down
   B) By measuring the relative speed of two clocks, you can determine which one is actually moving
   C) There are no rigid objects
   D) No object can move faster than light
   E) Massless objects always move at the speed of light

5. The fundamental reason large nuclei tend to α-decay is because
   A) There are a lot of protons repelling each other
   B) The α-particle isn't affected much by the strong force
   C) The nuclear force becomes repulsive at short range, forcing the α-particle out
   D) There are so many electrons that they electrostatically pull the α-particle out
   E) The α-particle is so large it can't fit inside the nucleus

6. According to the uncertainty principle, there is a lower limit on the ________ of the uncertainties of the position Δx and the momentum Δp.

   A) Sum B) Difference C) Average D) Ratio **E) Product**

7. According to general relativity, what effect does the *rotation* of the Earth have on objects near it?

   A) It pulls spacetime around with it in an effect called *frame dragging*
   B) It is the reason that orbits precess, rather than moving in constant elliptical orbits
   C) It causes gravitational radiation that makes the orbits slowly decay
   D) It is the *cause* of gravity; if the Earth weren't rotating, it would have no gravity
   E) None; as long as the Earth is spherical, it doesn't matter if it is rotating

8. A nucleus with four neutrons and three protons would be (Li: *Z*=3, Be: *Z*=4, N: *Z*=7)

   A) ^{4}Li **B) ^{7}Li** C) ^{3}Be D) ^{7}Be E) ^{3}N

9. Which of the following properties is *not* currently explained by the standard model?

   A) How the electron gets its mass
   B) Why the *Z* and *W* bosons have mass
   C) How neutrinos get their mass
   D) Why quarks are confined inside baryons and mesons
   E) How action at a distance can occur in electromagnetism

10. Which of the following formulas gives the energy of one photon?

    A) $\hbar\omega$ B) h/p C) p/h D) $\hbar f$ E) $p^2/(2m)$

11. Which particles do <u>not</u> have wave properties, according to quantum mechanics?

    A) Photons B) Electrons C) Atoms D) Molecules **E) All particles have wave properties**

12. Which of the following is <u>false</u> about gravitational waves

    A) They have not yet been directly experimentally observed
    B) Direct detection of these waves involves measuring distances in perpendicular directions to incredible precision
    C) They have been indirectly detected by studying neutron star pairs (pulsars) orbiting each other
    D) They are produced by accelerating masses
    E) They represent real distortions of distance, not just forces

13. The name of the tensor that is proportional to the stress-energy tensor in general relativity is the _______ tensor

    A) Riemann B) Ricci C) Metric **D) Einstein** E) Carlson

14. Which of the following is the correct formula for the momentum operator in one dimension?

    A) $\frac{\hbar}{i}\frac{\partial}{\partial x}$ B) $m\frac{dx}{dt}$ C) $-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}$ D) $\frac{h}{\lambda}$ E) None of these

15. The most penetrating and dangerous type of radiation comes from which nuclear decay?
   A) α B) β^+ C) β^- **D) γ** E) electron capture

16. Which of the following is not an assumption of the Bohr model of hydrogen?
   A) The angular momentum of the electron is always a multiple of $\hbar$
   B) The electron orbits the nucleus in circular orbits
   C) When the electron falls from one level to another, a single photon carries off the energy
   D) The electron's position is represented not by a specific value, but by a wave function ψ
   E) Actually, all of these are assumptions of the Bohr model

17. The gluon is believed to be the particle responsible for which force?
   A) Weak B) Gravity C) Electric D) Magnetic **E) Strong**

18. If I am traveling to the right at speed 0.6*c* and I send a light beam to the left (in vacuum), how fast will it move according to a stationary observer?
   A) 0.4*c* B) 0.6*c* **C) *c*** D) 1.6*c* E) None of these

19. Which of the following wave functions, if valid between $-\infty$ and ∞, would be normalizable?
   A) $\psi = e^{ax}$ B) $\psi = e^{-ax}$ C) $\psi = Bx^2$ D) $\psi = e^{+Ax^2}$ **E) $\psi = e^{-Ax^2}$**

20. Quarks inside a Δ^{++} baryon have in addition to their flavor (up) and charge (+2/3), another property that makes them not identical called
   A) Isospin **B) Color** C) Mass D) Parity E) Hypercharge

21. The expectation value of the Hamiltonian will tell you the average value of the ________ if you were to measure a particle
   A) Position B) Mass **C) Energy** D) Time E) Momentum

22. Under what circumstances can you tell which of two events *A* and *B* occurred first according to all observers?
   A) As long as they are not at the same time as viewed by any observer
   B) When they are spacelike separated
   C) When they are timelike separated
   D) Only if they occur at the same place
   E) Never; it is always ambiguous

23. Which of the following particles is not a fermion?
   A) Electron B) Up quark **C) Photon** D) Neutrino 1 E) Bottom quark

24. A cube of dimension 1×1×1 is moving in the *x*-direction at speed such that $\gamma = 2$. What would be the dimensions of the cube as measured by someone not moving with it?
   A) ½×½ ×½ B) 1×½ ×½ **C) ½×1×1** D) 2×2×2 E) 2×1×1

25. Which of the following is approximately the size of an atom?
A) 10^{-8} m **B)** 10^{-10} m C) 10^{-12} m D) 10^{-14} m E) 10^{-16} m

Part II: Short answer (review material) [20 points] (10 points each)
Choose **two** of the following three questions and give a short answer (1-3 sentences)

26. In relativity, nothing moves faster than light, but surely you could at least *signal* faster than light simply by, say, pushing on a rigid pole and detecting it at some distant place. Comment.

There is no such thing as a rigid object in special relativity. In fact if you attempted this approach, the signal would only be transmitted at the speed of sound, which in ordinary materials is much slower than the speed of light, and even for extraordinary materials (like neutron star material) is slower than the speed of light.

27. Explain what Rutherford was able to measure and to deduce by scattering α-particles off of gold atoms.

By measuring the recoil, he could tell that the positive charge, and most of the mass (essentially all of it) was concentrated in a tiny region called the nucleus. He was eventually able to measure the size of this nucleus, and found it was typically only a few fm across.

28. If you have a potential for a particle that changes at a boundary, you often have to calculate the wave function on the two sides of the potential, and then join them together. What conditions do you place on how the wave functions on the two sides are joined?

In general, because Schrödinger's equation is a second order differential equation, the wave function must both be continuous and its first derivative must be continuous.

Part III: Short answer (new material) [30 points] (10 points each)

Choose **three** of the following four questions and give a short answer (1-3 sentences) .

29. Explain, if you know the mass, charge, and spin of a particle, you would find the corresponding numbers for its anti-particle. Do all particles have anti-particles? Are the anti-particles always different from the particles?

The anti-particle has the same mass and spin, but the charge would be opposite. All particles have anti-particles, but for some particles (always neutral), the anti-particle is the same as the particle.

30. What sort of nuclei are likely to undergo (a) β^- decay, (b) β^+ decay or electron capture, and (c) α decay.

Nuclei with too many neutrons tend to undergo β^- decay; those with too many protons undergo β^+ decay, where the perfect balance is about 50/50 neutrons/protons for light nuclei ($Z < 50$) and 60/40 for heavy nuclei ($Z > 200$). Heavy nuclei ($Z > 200$) also undergo α decay because of the repulsion of their large positive charge.

31. Name at least three conservation laws that are respected by all particle physics forces, and name one that is respected by strong and electromagnetic forces, but not weak forces.

All particle physics forces conserve energy, momentum, baryon number and electric charge. Strangeness is conserved by strong and electromagnetic forces, but the weak force can violate it.

32. According to Newton, two neutron stars would orbit each other in an elliptical pattern, and will do so forever. Explain at least two ways in which Einstein's theory of gravity disagrees with this prediction.

First of all, orbits are close to but not identical with an ellipse; the main difference is that the major axis of the ellipse slowly precesses (changes directions) over time. Secondly, due to the emission of gravitational waves, the system loses energy, causing the neutron stars to slowly move together and eventually merge.

Part IV: Calculation (review material) [40 points] (20 points each)

Choose **two** of the following three questions and perform the indicated calculations

33. In principle (probably not in practice), one way to make the J/ψ particle is to collide a neutron and an anti-neutron head on, each of them moving at $v = 2.38\times10^8$ m/s.

(a) What is the Lorentz factor γ for the neutron and anti-neutron?

They are moving at the same speed, so they have the same Lorentz factor, which is

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\left(\frac{2.38\times10^8 \text{ m/s}}{2.998\times10^8 \text{ m/s}}\right)^2}} = \frac{1}{\sqrt{1-0.794^2}} = 1.645.$$

(b) The neutron has a mass of $m = 940\ \text{MeV}/c^2$. What is the mass of the anti-neutron? What is the energy of each of these particles? What is the mass of the J/ψ particle?

The anti-neutron has the same mass at the neutron, $m = 940\ \text{MeV}/c^2$. The energy of each of these particles is

$$E_n = E_{\bar{n}} = \gamma mc^2 = 1.645(940 \text{ MeV}) = 1546 \text{ MeV}.$$

Now, these two particles combine to make the J/ψ particle, which must have energy

$$E_J = E_n + E_{\bar{n}} = 2(1546 \text{ MeV}) = 3092 \text{ MeV}.$$

Since the particles were colliding head on and had the same speed and same mass, they have equal and opposite momenta, so the J/ψ will have no momentum and not be moving. Hence it will be at rest, which means we can use the formula $E_J = m_J c^2$, so $m_J = 3092\ \text{MeV}/c^2$. The actual mass is 3097, and the error is due to rounding.

(c) A neutron has a mean lifetime in its own frame of 882 s. What is the mean lifetime of these moving neutrons, and how far can they go before they decay?

An object moving last longer than when it is at rest, according to the formula $\Delta t = \gamma\tau$, so we have

$$\Delta t = \gamma\tau = 1.645(882 \text{ s}) = 1451 \text{ s}.$$

The we simply use $d = v\Delta t$ to find

$$d = v\Delta t = (2.38\times10^8 \text{ m/s})(1451 \text{ s}) = 3.45\times10^{11} \text{ m}.$$

34. A particle has wave function $\psi(x) = \begin{cases} N\sqrt{a^2x - x^3} & 0 \le x \le a, \\ 0 & \text{otherwise,} \end{cases}$

where *N* is an unknown normalization constant.

(a) What is the value of the constant *N*?

The wave function must be normalized, so the integral of the square of the magnitude is equal to one. We therefore have

$$1 = \int_{-\infty}^{\infty} \left|\psi(x)\right|^2 dx = \int_0^a N^2 \left(\sqrt{a^2x - x^3}\right)^2 dx = N^2 \int_0^a \left(a^2x - x^3\right) dx$$

$$= N^2 \left(\tfrac{1}{2}a^2x^2 - \tfrac{1}{4}x^4\right)\Big|_0^a = N^2\left(\tfrac{1}{2}a^4 - \tfrac{1}{4}a^4\right) = \tfrac{1}{4}N^2a^4,$$

$$N = \sqrt{\frac{4}{a^4}} = \frac{2}{a^2}.$$

(b) What is the most probable place(s) to find the particle?

To find the most probably place, we can find the largest value of the wave function magnitude or its square. In this case, it's easier to use the square, so we take the derivative and set it to zero to yield

$$0 = \frac{d}{dx}\left|\psi(x)\right|^2 = N^2 \frac{d}{dx}\left(a^2x - x^3\right) = N^2\left(a^2 - 3x^2\right),$$

$$3x^2 = a^2,$$

$$x = \frac{a}{\sqrt{3}}.$$

We keep only the positive square root because the other one is not in the allowed region.

(c) What is the probability that the particle is in the region $0 < x < \tfrac{1}{2}a$ **?**

We simply repeat the integral of part (a), but change the limits and substitute the value of the normalization constant from part (a).

$$P\left(0 < x < \tfrac{1}{2}a\right) = \int_0^{\frac{1}{2}a} \left|\psi(x)\right|^2 dx = N^2 \int_0^{\frac{1}{2}a} \left(a^2x - x^3\right) dx = \left(\frac{2}{a^2}\right)^2 \left(\tfrac{1}{2}a^2x^2 - \tfrac{1}{4}x^4\right)\Big|_0^{\frac{1}{2}a}$$

$$= \frac{4}{a^4}\left(\frac{1}{2}\frac{a^4}{4} - \frac{1}{4}\frac{a^4}{16}\right) = 4\left(\frac{1}{8} - \frac{1}{64}\right) = \frac{1}{2} - \frac{1}{16} = \frac{7}{16} = 43.75\%.$$

35. Tin is illuminated with a frequency of $f = 1.47\times10^{15}\ \text{s}^{-1}$. The tin is found to emit electrons which can overcome a potential of up to $V_{\text{max}} = 1.66\ \text{V}$.

(a) What is the work function for tin?

We use the equation for the photoelectric effect, $eV_{\text{max}} = hf - \phi$, and solve for the work function ϕ:

$$\begin{aligned}\phi = hf - eV_{\text{max}} &= \left(4.136\times10^{-15}\ \text{eV}\cdot\text{s}\right)\left(1.47\times10^{15}\ \text{s}^{-1}\right) - e\left(1.66\ \text{V}\right)\\ &= \left(6.08\ \text{eV}\right) - \left(1.66\ \text{eV}\right) = 4.42\ \text{eV}.\end{aligned}$$

(b) Suppose the light was changed to a wavelength of $\lambda = 153\ \text{nm}$. What would be the maximum voltage that could be overcome?

We need to first calculate the frequency, which is given by $\lambda f = c$, so then

$$f = \frac{c}{\lambda} = \frac{2.998\times10^{8}\ \text{m/s}}{153\times10^{-9}\ \text{m}} = 1.96\times10^{15}\ \text{s}^{-1}.$$

We then substitute it into the photoelectric effect equation to find

$$eV_{\text{max}} = hf - \phi = \left(4.136\times10^{-15}\ \text{eV}\cdot\text{s}\right)\left(1.96\times10^{15}\ \text{s}^{-1}\right) - \left(4.42\ \text{eV}\right) = 3.68\ \text{eV}.$$

$$V_{\text{max}} = 3.68\ \text{V}.$$

(c) For what frequency would the electrons be able to overcome a potential of $V_{\text{max}} = 2.32\ \text{V}$?

This time we solve for *f*, which yields

$$hf = eV_{\text{max}} + \phi = e\left(2.32\ \text{V}\right) + \left(4.42\ \text{eV}\right) = 6.74\ \text{eV},$$

$$f = \frac{6.74\ \text{eV}}{h} = \frac{6.74\ \text{eV}}{4.136\times10^{-15}\ \text{eV}\cdot\text{s}} = 1.63\times10^{15}\ \text{s}^{-1}.$$

(d) What is the longest wavelength that can remove an electron from tin?

The lowest frequency corresponds to the frequency where the energy of the photon equals the work function, so $hf = \phi$. The wavelength is given by $\lambda f = c$, so solving for λ, we have

$$\lambda = \frac{c}{f} = \frac{c}{\phi/h} = \frac{hc}{\phi} = \frac{\left(4.136\times10^{-15}\ \text{eV}\cdot\text{s}\right)\left(2.998\times10^{8}\ \text{m/s}\right)}{4.42\ \text{eV}} = 2.81\times10^{-7}\ \text{m} = 281\ \text{nm}.$$

Part V: Calculation (new material): [60 points]
Choose **three** of the following four questions and perform the calculations (20 points each)

36. One of the rarer isotopes of potassium is ^{40}K. A certain sample of ^{40}K has a mass of 4.02×10^{-3} g, and undergoes approximately 1038 decays every second

(a) What is the approximate mass in u of one atom of ^{40}K? How many atoms of ^{40}K do we have?

The mass number gives you the approximate mass of ^{40}K, so it is about 40 u. The number of atoms is therefore

$$N=\frac{M}{m}=\frac{4.02\times10^{-3}\text{ g}}{40\text{ u}}=\frac{4.02\times10^{-3}}{40}N_A=\frac{\left(4.02\times10^{-3}\right)\left(6.022\times10^{23}\right)}{40}=6.052\times10^{19}\,.$$

(b) What is the decay rate λ in s^{-1}? What is the half-life in y? ($\text{y}=3.156\times10^{7}\text{ s}$)

We use the formula $R=\lambda N$ and solve for N, so we have

$$\lambda=\frac{R}{N}=\frac{1038\text{ s}^{-1}}{6.052\times10^{19}}=1.715\times10^{-17}\text{ s}^{-1}=\left(1.715\times10^{-17}\text{ s}^{-1}\right)\left(3.156\times10^{7}\text{ s/y}\right)=5.412\times10^{-10}\text{ y}^{-1}\,.$$

The half-life is related to this by $\lambda t_{1/2}=\ln(2)$, so we have

$$t_{1/2}=\frac{\ln(2)}{\lambda}=\frac{0.6931}{5.413\times10^{-10}\text{ y}^{-1}}=1.281\times10^{9}\text{ y}=1.281\text{ Gy}\,.$$

(c) The Earth is 4.56×10^{9} y old. Assuming this sample has been around that whole time, how many ^{40}K atoms were there to begin with? What was their total mass in g?

Due to radioactive decay, the number of particles should be decaying exponentially, so we have $N=N_0e^{-\lambda t}$. We therefore have

$$N_0=Ne^{\lambda t}=\left(6.052\times10^{19}\right)\exp\left[\left(5.412\times10^{-10}\text{ y}^{-1}\right)\left(4.56\times10^{9}\text{ y}\right)\right]=6.052\times10^{19}e^{2.468}$$
$$=7.140\times10^{20}\,.$$

We then can simply multiply by the mass of each atom again, to get

$$M_0=N_0m=\left(7.140\times10^{20}\right)\left(40\text{ u}\right)=\frac{2.856\times10^{22}\text{ u}}{6.022\times10^{23}\text{ u/g}}=0.04742\text{ g}\,.$$

37. On the nest page is a list of isotopes and their masses. ^{108}Ag is a nucleus that might decay by one of the modes listed at right. You may use the table at right to summarize your answers.

(a) For each of these, what is the daughter isotope?

mode	daughter	Q (MeV)	?
α	^{104}Rh	– 3.077	no
electron capture	^{108}Pd	1.919	yes
β^+	^{108}Pd	0.897	yes
β^-	^{108}Cd	1.649	yes

For α-decay, Z should decrease by two and A by four, so the daughter is ^{104}Rh. For electron capture and β^+ decay, since we are losing one unit of charge while not changing A, so we have ^{108}Cd. Finally for β^- decay, we are losing a negative unit of charge, so we have ^{108}Pd. These have been included in the table.

(b) What is the *Q*-value for each decay?

For α-decay, the formula for Q is $Q = \left(M_P - M_D - M_{4He}\right)c^2$, so we have

$$Q = \left(M_P - M_D - M_{4He}\right)c^2 = \left(107.905953 - 103.906654 - 4.002602\right)\text{u}c^2$$
$$= \left(-0.003303\right)\left(931.5\text{ MeV}\right) = -3.077\text{ MeV}.$$

For electron capture the formula is simply $Q = \left(M_P - M_D\right)c^2$, so

$$Q = \left(M_P - M_D\right)c^2 = \left(107.905953 - 107.903893\right)\text{u}c^2 = \left(0.002060\right)\left(931.5\text{ MeV}\right) = 1.919\text{ MeV}$$

For β^+ decay, it is the same daughter, but the formula changes to $Q = \left(M_P - M_D\right)c^2 - 2m_ec^2$, so

$$Q = \left(M_P - M_D\right)c^2 - 2m_ec^2 = \left(1.919\text{ MeV}\right) - \left(1.022\text{ MeV}\right) = 0.897\text{ MeV}.$$

Finally, for β^- decay, we again have $Q = \left(M_P - M_D\right)c^2$, but the daughter isotope has changed.

$$Q = \left(M_P - M_D\right)c^2 = \left(107.905953 - 107.904183\right)\text{u}c^2 = \left(0.001770\right)\left(931.5\text{ MeV}\right) = 1.649\text{ MeV}.$$

All of these values have been included in the table.

(c) Which of the modes can actually occur?

Only those with positive energy actually work, so electron capture, β^+ decay, and β^- decay are all possible, but α decay is not. Also included in the table.

38. There is a particle Ξ^{*-} which decays by strong interactions as follows: $\Xi^{*-} \rightarrow \Sigma^- + X$. The Ξ^{*-} and Σ^- are both baryons, and the other particles in the table at right are all mesons. The spin and strangeness of the other particles are listed at right, and the charges are implied by their names.

All masses in MeV/c^2

Name	Mass	Spin	Strange
Ξ^{*-}	1820	3/2	–2
Σ^-	1197	1/2	–1
π^0	135	0	0
K^-	494	0	–1
K^0	498	0	+1
$\bar{K}^{*0}$	892	1	–1

(a) What is the charge and strangeness of the *X* particle?

Charge is always conserved, and since strong interactions conserve strangeness, strangeness is also conserved. We solve for the charge by demanding the two sides have equal charge, so $-1 = -1 + q$, so $q = 0$. Similarly for strangeness, $-2 = -1 + s$, so $s = -1$. In summary,

$$q = 0 \quad \text{and} \quad s = -1.$$

(b) Is it a baryon, anti-baryon, or a meson?

Baryon number is conserved, and since there is already one baryon on each side of this equation, the *X* must have baryon number zero. Only mesons have baryon number zero and have strong interactions, so it must be a meson.

(c) What, if anything, can you conclude about the mass of the *X*?

Because this is a decay, the mass of the particle on the left must exceed the sum of the masses on the right, that is, $m_{\Xi^*} > m_\Sigma + m_X$, or rearranging,

$$+m_X < m_{\Xi^*} - m_\Sigma = \left(1820 \text{ MeV}/c^2\right) - \left(1197 \text{ MeV}/c^2\right) = 623 \text{ MeV}/c^2 .$$

(d) Could the *X* be any of the particles in the table given? Could it be any of their anti-particles?

We need a meson with charge 0, strangeness –1 and mass less than 623 MeV/c^2. No such particle is in the table. However, the K^0 satisfies all of these except it has the wrong sign for the strangeness, so its anti-particle the $\bar{K}^0$ would work, and in fact this is exactly the particle that results from this decay.

39. A neutron star of radius $R = 11.5\text{ km}$ is being studied by a researcher. She is far from the neutron star, and discovers that a spectral line normally at 582 nm is observed, by her, at a wavelength of 762 nm.

(a) What is the mass of the neutron star, in solar masses ($M_\odot = 1.989 \times 10^{30}\text{ kg}$)?

The normal spectral line $\lambda_0 = 582\text{ nm}$ has been stretched to $\lambda = 762\text{ nm}$ by gravitational red-shift, so rearranging the equation for gravitational red-shift, we have

$$\sqrt{1 - \frac{2GM}{c^2 r}} = \frac{\lambda_0}{\lambda} = \frac{582\text{ nm}}{762\text{ nm}} = 0.764,$$

$$\frac{2GM}{c^2 r} = 1 - 0.764^2 = 0.417,$$

$$\frac{2GM}{c^2} = 0.417r = 0.417(11.5\text{ km}) = 4.79\text{ km} = 4790\text{ m},$$

$$M = \frac{c^2}{2G}(4790\text{ m}) = \frac{\left(2.998 \times 10^8\text{ m/s}\right)^2 (4790\text{ m})}{2\left(6.674 \times 10^{-11}\text{ m}^3\text{kg}^{-1}\text{s}^{-2}\right)} = 3.23 \times 10^{30}\text{ kg},$$

$$M = \frac{3.23 \times 10^{30}\text{ kg}}{1.989 \times 10^{30}\text{ kg}/M_\odot} = 1.62 M_\odot.$$

(b) If she were to travel to a distance $R = 27.0\text{ km}$ from the center of the neutron star, and spent 5.00 days doing research there (as measured by her), how much time would pass according to an observer far from the neutron star?

The proper time is the time measured by her, $\tau = 5.00\text{ day}$. We therefore use the relationship between the external time and the proper time which is

$$t = \frac{\tau}{\sqrt{1 - \frac{2GM}{c^2 r}}} = \frac{5\text{ day}}{\sqrt{1 - \frac{4.79\text{ km}}{27.0\text{ km}}}} = 5.513\text{ days}.$$

We took advantage of the fact that we already had $2GM/c^2$ from part (a).

(c) To what radius would this neutron star have to shrink to become a black hole?

The work was all done in part (a), so we proceed immediately to the answer,

$$R_S = \frac{2GM}{c^2} = 4790\text{ m} = 4.79\text{ km}.$$

Equations

Constants:

$$h = 6.626\times10^{-34}\ \text{J}\cdot\text{s} = 4.136\times10^{-15}\ \text{eV}\cdot\text{s}$$
$$\hbar = 1.055\times10^{-34}\ \text{J}\cdot\text{s} = 6.582\times10^{-16}\ \text{eV}\cdot\text{s}$$
$$N_A = 6.022\times10^{23}$$
$$G = 6.674\times10^{-11}\ \text{m}^3\text{kg}^{-1}\text{s}^{-2}$$

$$\text{u} = 931.5\ \text{MeV}/c^2 = 1.661\times10^{-27}\ \text{kg}$$
$$2m_e c^2 = 1.022\ \text{MeV}$$
$$M_{\text{He}} = 4.002602\ \text{u}$$

Photoelectric effect: $eV_{\max} = hf - \phi$

Gravitational time dilation and red shift: $\tau = t\sqrt{1-\dfrac{2GM}{c^2 r}}$, $\lambda = \dfrac{\lambda_0}{\sqrt{1-\dfrac{2GM}{c^2 r}}}$

Schwarzschild radius: $R_S = \dfrac{2GM}{c^2}$

Isotope Masses

Z	Element	Symbol	A	Atomic Mass
45	Rhodium	Rh	102	101.906794
			103	102.905502
			104	103.906654
46	Palladium	Pd	102	101.905616
			104	103.904033
			105	104.905082
			106	105.903481
			107	106.905126
			108	107.903893
			110	109.905158
47	Silver	Ag	107	106.905091
			108	107.905953
			109	108.904754
			110	109.906110
48	Cadmium	Cd	106	105.906457
			108	107.904183
			109	108.904984
			110	109.903004
			111	110.904182
			112	111.902760
			113	112.904401
			114	113.903359
			116	115.904755
49	Indium	In	113	112.904060
			114	113.904916
			115	114.903876
			116	115.905258

Z	Element	Symbol	A	Atomic Mass