

Name _____

Test 2 October 19, 2018

This test consists of three parts. Please note that in parts II and III, you can skip one question of those offered. The equations below may be helpful with some problems.

<p style="text-align: center;"><u>Constants</u></p> $h = 6.626 \times 10^{-34} \text{ J}\cdot\text{s}$ $h = 4.136 \times 10^{-15} \text{ eV}\cdot\text{s}$ $\hbar = 1.055 \times 10^{-34} \text{ J}\cdot\text{s}$ $\hbar = 6.582 \times 10^{-16} \text{ eV}\cdot\text{s}$ $k_B = 1.3807 \times 10^{-23} \text{ J/K}$ $k_B = 8.6173 \times 10^{-5} \text{ eV/K}$ $k = 8.988 \times 10^9 \text{ N}\cdot\text{m}^2 / \text{C}^2$ $e = 1.602 \times 10^{-19} \text{ C}$ $1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$ $\alpha = \frac{ke^2}{\hbar c} \approx \frac{1}{137}$	<p style="text-align: center;"><u>Hydrogen-Like Atoms</u></p> $E = -\frac{k^2 e^4 \mu Z^2}{2\hbar^2 n^2} = -\frac{(\mu c^2) \alpha^2 Z^2}{2n^2}$ $E = \frac{-(13.60 \text{ eV}) Z^2}{n^2}$	<p style="text-align: center;"><u>Hydrogen Spectrum</u></p> $\lambda = (91.17 \text{ nm}) \left(\frac{1}{n^2} - \frac{1}{m^2} \right)^{-1}$
<p style="text-align: center;"><u>Wave Relationships</u></p> $\lambda = \frac{2\pi}{k}$ $\frac{\omega}{2\pi} = f = \frac{1}{T}$	<p style="text-align: center;"><u>Reduced Mass</u></p> $\mu = \frac{mM}{m+M}$	<p style="text-align: center;"><u>Black Bodies</u></p> $U = \frac{\pi^2 (k_B T)^4}{15(\hbar c)^3}$ $\lambda_{\text{max}} T = .002898 \text{ m}\cdot\text{K}$
	<p style="text-align: center;"><u>Compton Effect</u></p> $\lambda' - \lambda = \frac{h}{mc} (1 - \cos \theta)$ $\frac{h}{mc} = 2.426 \times 10^{-12} \text{ m}$	<p style="text-align: center;"><u>Rutherford Scattering</u></p> $b = \frac{kqQ}{m_\alpha v^2} \cot\left(\frac{\theta}{2}\right)$ $R = \frac{2Ze^2 k}{E}$

Part I: Multiple Choice [20 points]

For each question, choose the best answer (2 points each)

1. The ratio of the charge to mass of an electron was measured by J. J. Thomson by measuring
 - A) How the electron curved in the presence of an electric field
 - B) How the electron curved in the presence of a magnetic field
 - C) The scattering of alpha-particles off of nuclei
 - D) The change in mass of an oil droplet as it picked up or lost an electron
 - E) The quantity of charge required to dissociate a mole of various compounds in solution

2. In the photoelectric effect, which determines how much energy an electron ejected from the metal has?
 - A) The polarization of the light
 - B) The intensity (brightness) of the light
 - C) The direction the light impinges the metal at
 - D) The frequency of the light
 - E) The size of the electric field in the light

3. The speed at which a wave packet as a whole moves is called the _____ velocity, while the speed at which the peak of an individual wave moves is called the _____ velocity
 - A) Group, phase
 - B) Phase, group
 - C) Packet, peak
 - D) Lump, wave
 - E) Bunch, wave

4. Which of the following equals $\cos \theta - i \sin \theta$?
 A) $e^{i\theta}$ B) $-e^{i\theta}$ C) $e^{-i\theta}$ D) $-e^{-i\theta}$ E) None of these
5. Wave properties of objects passing through an aperture (like a slit) are most likely to be noticeable when
 A) The wavelength is large compared to the aperture
 B) The wavelength is small compared to the aperture
 C) The frequency is much lower than the speed of the particles
 D) The frequency is much higher than the speed of the particles
 E) The waves are generated by a wave generator, rather than a particle generator.
6. The de Broglie relationship between wavelength and momentum is
 A) $p = h\lambda$ B) $\lambda = hp$ C) $\lambda hp = 1$ D) $\lambda p = h$ E) None of these
7. Find the partial derivative $\frac{\partial}{\partial x}(x^2 - y^2)$
 A) $2x - 2y$ B) $2x - y^2$ C) $-2y$ D) x^2 E) $2x$
8. In which situation is the Bohr model not successful in describing the energy levels?
 A) An hydrogen atom
 B) A non-hydrogen atom with one electron
 C) A many-electron atom describing the innermost electrons
 D) A many-electron atom describing the outermost electrons
 E) Actually, it applies to all of these cases
9. Which of the following are believed to not have quantum mechanical properties, that is, they do not have wave-like properties
 A) Electrons
 B) Protons
 C) Atoms
 D) Molecules
 E) They actually all have wave properties
10. How did the scattering measurements of Rutherford of alpha-particle off of atoms change our picture of the atom?
 A) We realized that alpha-particles were in fact, very large, rather than very small
 B) We learned that electrons are not present in the nucleus of the atom
 C) We learned that the positive charge is concentrated in a small region called the nucleus
 D) We learned that the electrons are attracted to the nucleus
 E) We learned the approximate size of atoms

Part III: Calculation: [60 points]

Choose **three** of the following four questions and perform the indicated calculations (20 points each).

14. In the early universe, there was a time when protons and electrons first came together to form hydrogen atoms. This era is called *recombination*, and it occurred at a temperature of approximately $T = 2976$ K. It is believed that the radiation at this time was almost a perfect black body radiation
- (a) What wavelength of light λ_{\max} would have been the peak of the spectrum at this time?
 - (b) What is the corresponding frequency (in Hz) and energy (in eV) for a photon with the wavelength you found in (a)?
 - (c) What was the energy density (in J/m^3) of the black-body radiation at that time?
15. A tau-minus lepton (τ^-) is much like an electron, but much heavier. It has a mass of $m_\tau = 1776.8 \text{ MeV}/c^2$, and can bind to a proton with a mass of $m_p = 938.3 \text{ MeV}/c^2$.
- (a) What is the reduced mass for this system?
 - (b) Find a formula for the energy of the n 'th state for this system, in keV.
 - (c) An atom in this system goes from the $n = 25$ state to the $n = 24$ state. What is the energy in eV of the photon released in the process?
16. An X-ray photon with unknown wavelength scatters from a stationary target. It is found that for those X-rays that are scattered at an angle of 60° , the scattered X-rays have a wavelength of $14.74 \text{ pm} = 14.74 \times 10^{-12} \text{ m}$
- (a) What was the wavelength of the photons before they were scattered?
 - (b) What is the wavelength of the photons that are scattered by 180° ?
 - (c) At what angle θ will the scattered wavelength be $17.10 \text{ pm} = 17.10 \times 10^{-12} \text{ m}$?
17. The wave function for a particle is given by $\psi(x) = \begin{cases} N(e^{-\lambda x} - e^{-2\lambda x}) & x \geq 0, \\ 0 & x < 0, \end{cases}$ where λ is a positive constant. A possibly helpful integral is given below.
- (a) Where in the allowed region $x \geq 0$ is the particle most likely to be?
 - (b) What is the normalization constant N ?
 - (c) If the particle's position is measured, what is the probability that it will have $x > \lambda^{-1}$?

Possibly helpful integral: $\int_{\alpha}^{\infty} e^{-Ax} dx = e^{-A\alpha} / A$