Name ___________________

Test 3
November 7, 2018

This test consists of three parts. Please note that in parts II and III, you can skip one question of those offered. Some possibly useful formulas can be found below.

$h = 6.626\times10^{-34}\ \text{J}\cdot\text{s} = 4.136\times10^{-15}\ \text{eV}\cdot\text{s}$

$\hbar = 1.055\times10^{-34}\ \text{J}\cdot\text{s} = 6.582\times10^{-16}\ \text{eV}\cdot\text{s}$

$1\ \text{eV} = 1.602\times10^{-19}\ \text{J}$

Barrier penetration:

$$T \approx 16\frac{E}{V_0}\left(1-\frac{E}{V_0}\right)e^{-2\alpha L}$$

$$\alpha = \frac{\sqrt{2m(V_0-E)}}{\hbar}$$

Hydrogen

$$E_n = -\frac{(13.6\ \text{eV})Z^2}{n^2}$$

Reflection off a step:

$$R = \begin{cases} \left(\dfrac{\sqrt{E}-\sqrt{E-V_0}}{\sqrt{E}+\sqrt{E-V_0}}\right)^2 & \text{if } E > V_0 \\ 1 & \text{if } E < V_0 \end{cases}$$

Euler's formula

$$e^{ix} = \cos x + i\sin x$$

Spherical Coords.

$x = r\sin\theta\cos\phi$

$y = r\sin\theta\sin\phi$

$z = r\cos\theta$

$0 \le r < \infty$

$0 \le \theta \le \pi$

$0 \le \phi < 2\pi$

Part I: Multiple Choice [20 points]
For each question, choose the best answer (2 points each)

1. If the total angular momentum squared has a value of $L^2 = 20\hbar^2$, what is l?
   A) 4 B) 5 C) 20 D) 380 E) 420

2. An electron with $n = 4$, $l = 2$, and $m = 1$ would be described as what type of electron?
   A) 4p B) 2p C) 2f D) 4f E) 4d

3. Which of the following tells you the probability density of finding a particle at the point $\vec{r}$ at time t in three dimensions?
   A) $\psi(\vec{r})$ B) $\psi^*(\vec{r})$ C) $[\psi(\vec{r})]^2$ D) $|\psi(\vec{r})|$ E) $|\psi(\vec{r})|^2$

4. If an electron in an atom has $l = 2$, what is a complete list of the values that m can take on?
   A) ±2 B) $\pm2,\pm1,0$ C) $0,1,2$ D) $\pm2,\pm\frac{3}{2},\pm1,\pm\frac{1}{2},0$ E) None of these

5. Which of the following corresponds to the momentum operator p_{op} in quantum mechanics in one dimension?
   A) $\frac{\hbar}{i}\frac{\partial}{\partial x}$ B) $-\frac{\hbar}{i}\frac{\partial}{\partial x}$ C) x D) $\hbar x$ E) none of these

6. How come we don't solve Schrödinger's equation for hydrogen by factoring the wave function into functions of *x*, *y*, and *z*: $\Psi(\vec{r}) = X(x)Y(y)Z(z)$?
   A) Because the derivative terms end up mixing these factors together into an intractable mess
   B) Because there are three Schrödinger's equations in 3D, and this can't solve all three at once
   C) Because it should be a *sum* of functions in 3D, not a product
   D) Because the potential for hydrogen can't be naturally written in Cartesian coordinates, and this factorization doesn't help
   E) It can be written this way; that's exactly how we solved it

7. To find the average energy you would expect if you measured a particle, you should calculate the expectation value of the
   A) Position B) Momentum C) Momentum squared D) Hamiltonian E) None of these

8. Suppose $\Psi_1(\vec{r},t)$ and $\Psi_2(\vec{r},t)$ are both solutions of Schrödinger's time-dependent equation. Which of the following is guaranteed to also be a solution?
   A) $\Psi_1^*(\vec{r},t)+\Psi_2^*(\vec{r},t)$
   B) $\Psi_1(\vec{r},t)\cdot\Psi_2(\vec{r},t)$
   C) $\Psi_1(\vec{r},t)/\Psi_2(\vec{r},t)$
   D) $\Psi_1(\vec{r},t)-\Psi_2(\vec{r},t)$
   E) None of these

9. For the harmonic oscillator with potential $V(x)=\frac{1}{2}m\omega^2x^2$, why are there only bound states, no unbound states?
   A) Because the force never vanishes, the particle must always be bound
   B) Because e^{ikx} is not a solution to Schrödinger's equation, there can't be unbound states
   C) Because the potential at infinity is infinity, the energy can't exceed the potential there
   D) Because all solutions have positive energy, there can't be unbound states
   E) They both exist, we just only found the unbound states

10. When you try to penetrate a finite-thickness barrier of size *L*, and your energy *E* is smaller than the height of the barrier, how does the barrier thickness affect the probablity of getting through?
   A) The probability grows exponentially as the thickness increases
   B) The probability shrinks exponentially as the thickness increases
   C) The probability is inversely proportional to the thickness
   D) The probability is inversely proportional to the thickness squared
   E) The probability is always zero in this case

Part II: Short answer [20 points]

Choose two of the following three questions and give a short answer (2-4 sentences) (10 points each).

11. Suppose you have found a solution $\psi(x)$ to Schrödinger's time-independent equation. Unfortunately, it turns out that the function is not normalized probably. What should you do? You will need to include one or more formulas in your answer.

12. When solving Schrödinger's equation, it is often necessary to solve it in different regions, and then piece the potential together at the boundaries. What are appropriate boundary conditions if (i) you have solutions on both sides of the boundary, or (ii) the potential is infinite on one side of the boundary.

13. According to our computations for hydrogen, the 3p, 3d, and 4s energies are related by $E_{3p} = E_{3d} < E_{4s}$. Explain qualitatively why this doesn't work for more complicated atoms, and give the correct general heirarchy for these levels, according to our more sophisticated model.

Part III: Calculation: [60 points] Choose three of the following four questions and perform the indicated calculations (20 points each).

14. An atom contains a single electron in the state $n = 6$, but the nuclear charge is unknown. The amount of energy required to extract the electron from the atom is measured to be $E = 3.40\ \text{eV}$
    (a) What is the value of the nuclear charge Z?
    (b) If the atom were to emit one photon, starting in this state, what would be the smallest energy E that that emitted photon could have, and what would be the final value of n?
    (c) If the atom were to absorb one photon, starting in this state, what would be the smallest energy E that that absorbed photon could have, and what would be the final value of n?

15. A group of electrons with kinetic energy $E = 8.00\ \text{eV}$ impacts a sudden step potential of unknown height V_0. It is found that of 9.00×10^6 electrons, 8.00×10^6 of them <u>successfully</u> penetrate the barrier.
    (a) What is the <u>reflection</u> probability R?
    (b) What is the value (or possible values) of the potential height V_0?

16. A particle has wave function $\psi(x) = \begin{cases} 2\lambda^{3/2} x e^{-\lambda x} & \text{for } x > 0, \\ 0 & \text{for } x < 0. \end{cases}$

    This wave function is properly normalized and has $\langle p^2 \rangle = \hbar^2 \lambda^2$. A possibly useful integral is below.
    (a) Find the expectation values of $\langle p \rangle$, $\langle x \rangle$ and $\langle x^2 \rangle$.
    (b) Find the uncertainties Δx and Δp, and check that it satisfies the uncertainty principle.

Possibly useful integral: $\int_0^\infty x^n e^{-Ax} dx = n! / A^{n+1}$.

17. Chemists prefer using real wave functions for atoms. For example, one of the states they use is $\psi = \frac{1}{\sqrt{2}}\left(\psi_{2,1,-1} - \psi_{2,1,+1}\right)$. Some hydrogen wave functions are written below.
    (a) Write explicitly this wave function ψ as a function of (r,θ,ϕ), and show that it is real.
    (b) Find the value(s) of (r,θ,ϕ) where the particle is most likely to be.
    (c) Find the corresponding cartesian coordinates (x,y,z).

$$R_{2,1} = \frac{r}{2\sqrt{6a^5}} e^{-r/2a}, \quad Y_{1,\pm1} = \mp\sqrt{\frac{3}{8\pi}} \sin\theta e^{\pm i\phi}, \quad Y_{2,\pm1} = \mp\sqrt{\frac{15}{8\pi}} \sin\theta \cos\theta e^{\pm i\phi}.$$