

Test 3    Name \_\_\_\_\_  
November 3, 2021

This test consists of three parts. Please note that in parts II and III, you can skip one question of those offered. Some possibly useful formulas can be found below.

$h = 6.626 \times 10^{-34} \text{ J} \cdot \text{s} = 4.136 \times 10^{-15} \text{ eV} \cdot \text{s}$ $\hbar = 1.055 \times 10^{-34} \text{ J} \cdot \text{s} = 6.582 \times 10^{-16} \text{ eV} \cdot \text{s}$ $1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$	<b>Barrier penetration:</b> $T \approx 16 \frac{E}{V_0} \left(1 - \frac{E}{V_0}\right) e^{-2\alpha L}$ $\alpha = \sqrt{2m(V_0 - E)}/\hbar$	<b>1D square well:</b> $E_n = \frac{\pi^2 \hbar^2 n^2}{2mL^2}$ $\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi nx}{L}\right)$ $n = 1, 2, 3, \dots$
<b>Reflection off a step:</b> $R = \begin{cases} \left(\frac{\sqrt{E} - \sqrt{E - V_0}}{\sqrt{E} + \sqrt{E - V_0}}\right)^2 & \text{if } E > V_0 \\ 1 & \text{if } E < V_0 \end{cases}$	<b>Harmonic Oscillator</b> $E_n = \hbar\omega\left(n + \frac{1}{2}\right)$ $n = 0, 1, 2, \dots$	<b>Hydrogen</b> $E_n = -\frac{(13.6 \text{ eV})Z^2}{n^2}$

**Part I: Multiple Choice [20 points]**

For each question, choose the best answer (2 points each)

1. Suppose that a single photon is sent through a half-silvered mirror, so that the wave function is reflected 50% of the time and transmitted 50% of the time. We then place detectors to see which way it goes. Assuming the detectors are perfect, what is the probability that *neither* of them will see the photon?  
 A) 0%            B) 25%            C) 50%            D) 100%            E) None of these
  
2. For which of the following problems is the energy spacing between level  $n$  and level  $n+1$  always the same, independent of  $n$ ?  
 A) Hydrogen atom  
 B) Infinite square well  
 C) Finite square well  
 D) Harmonic oscillator  
 E) None of the above
  
3. Suppose you solve Schrodinger's equation in two regions. What constraint(s) should you impose at the boundary between the two regions?  
 A) Match the wave function (only)  
 B) Match the derivative of the wave function (only)  
 C) Match BOTH the wave function and the derivative  
 D) Make sure both wave functions vanish at the boundary  
 E) Make sure both wave functions are finite at the boundary

4. Which of the following might be a good statement of the Pauli exclusion principle for electrons?
- Electrons repel each other, so they tend not to be near each other
  - If one electron is in the  $n$ 'th level of a hydrogen atom, no other electron can be in that level
  - Some levels of hydrogen-like atoms are excluded from containing an electron
  - Electrons are excluded from the nucleus, because it is already filled with protons and neutrons
  - Two electrons can never be in the same quantum state
5. If you have a 4d electron, what are the values of  $n$  and  $l$ ?
- $n = 3, l = 4$
  - $n = 4, l = 3$
  - $n = 2, l = 4$
  - $n = 4, l = 2$
  - None of these
6. Suppose you have a solution to Schrodinger's equation, but it is not properly normalized, so you find  $\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 4$ . What should you multiply  $\psi(x)$  by to fix this?
- $\frac{1}{4}$
  - $\frac{1}{2}$
  - $\frac{1}{16}$
  - 2
  - 4
7. Which of the following might represent the wave function for a particle moving in the  $+x$  direction?
- $e^{-ikx}$
  - $e^{ikx}$
  - $\cos(kx)$
  - $\sin(kx)$
  - $e^{-Ax^2}$
8. Which of the following is the momentum operator for a particle in one dimension?
- $\frac{\hbar}{i} \frac{d}{dx}$
  - $i\hbar \frac{d}{dx}$
  - $i\hbar x$
  - $-i\hbar x$
  - None of these
9. When we look at hydrogen wave functions, for which types of wave functions have a chance that the electron is near the origin?
- Those with small values of  $n$
  - Those with large values of  $n$
  - Those with small values of  $l$
  - Those with large values of  $l$
  - The electron can never be near the origin for any hydrogen wave function
10. When does the uncertainty relation  $(\Delta x)(\Delta p) \geq \frac{1}{2} \hbar$  **not** apply?
- Bound states
  - Unbound states
  - When the energy is positive
  - When the energy is negative
  - Never, it always applies

**Part II: Short answer [20 points]**

Choose two of the following three questions and give a short answer (2-4 sentences) (10 points each).

11. What sorts of potentials  $V(\mathbf{r}, t)$  in three dimensions have the property such that it makes sense to write solutions like  $\Psi(\mathbf{r}, t) = \psi(\mathbf{r})e^{-iEt/\hbar}$ , factoring it into a time component and a space component? For which sorts of potentials does it make more sense to write the space part in spherical coordinates as  $\psi(\mathbf{r}) = R(r)Y_{l,m}(\theta, \phi)$ ?
12. For a particle in one dimension with mass  $m$  and potential  $V(x)$ , what is the formula for the Hamiltonian? If I take a wave function  $\psi(x)$  and use it to calculate the expectation value of the Hamiltonian  $\langle H \rangle$ , what would this correspond to?
13. A particle of energy  $E$  impacts a step potential of size  $V_0$ . Explain if the wave is partly reflected, completely reflected, or completely transmitted in the four cases  $E < V_0$ ,  $0 < V_0 < E$ ,  $V_0 = 0$  and  $V_0 < 0$ . No equations are necessary.

**Part III: Calculation: [60 points]** Choose three of the following four questions and perform the indicated calculations (20 points each).

14. A particle has wave function given by  $\psi(x) = \begin{cases} 2\alpha^{3/2}xe^{-\alpha x} & \text{if } x > 0, \\ 0 & \text{if } x < 0. \end{cases}$ . This wave function is normalized. It has expectation value  $\langle p^2 \rangle = \hbar^2\alpha^2$ . A possibly useful integral appears below.
- Find the expectation values  $\langle x \rangle$ ,  $\langle x^2 \rangle$ , and  $\langle p \rangle$  for this wave function.
  - What are the uncertainties  $\Delta x$  and  $\Delta p$  for this wave function?
  - Check that it satisfies the uncertainty relation.

**Possibly useful integral:**  $\int_0^{\infty} x^n e^{-Ax} dx = \frac{n!}{A^{n+1}}$ , where  $n! = n(n-1)(n-2)\cdots 2 \cdot 1$ .

15. An electron with mass  $9.109 \times 10^{-31}$  kg is trapped in a 1D infinite square well of length  $L$ . It is in the state  $n = 5$  and is found to have an energy of  $1.50 \times 10^{-19}$  J
- What is the length  $L$ ?
  - What is the energy of the state  $n = 3$  for this potential?
  - If the electron starting in  $n = 5$  absorbed  $1.44 \times 10^{-19}$  J, what would be the final  $n$  value?
16. Electrons of mass  $m = 9.109 \times 10^{-31}$  kg  $= 5.11 \times 10^5$  eV/ $c^2$  have an energy of  $E = 3.00$  eV and impact a barrier of width  $L = 0.270$  nm  $= 2.70 \times 10^{-10}$  m and potential of height  $V_0 = 10.00$  eV.
- What is the damping coefficient  $\alpha$  in  $\text{m}^{-1}$  or  $\text{nm}^{-1}$ ?
  - What is the probability that the electrons manage to penetrate the barrier?
  - The barrier width is now increased, and it is now found that only  $4.27 \times 10^{-5}$  of the electrons make it through. What is the new width  $L$ ?
17. An electron in hydrogen is measured and found to have total orbital angular momentum  $L^2 = 12\hbar^2$ .
- What is the value of the angular momentum quantum number  $l$ ? What values for the  $z$ -component  $L_z$  are possible?
  - Suppose the *total* angular momentum around the  $z$ -axis were measured, defined as  $J_z = L_z + S_z$ . What is the largest (most positive) and smallest (most negative) values that  $J_z$  could be?
  - Suppose that  $J_z$  has the value  $J_z = +\frac{3}{2}\hbar$ . Argue that there are two possible values of the pairs  $L_z$  and  $S_z$  that allow this to happen.
  - Given the  $l$ -value you found in part (a), what is the lowest (most negative) energy value  $E_n$  that the atom could have?