

Name _____

Final Exam
December 10, 2018

This test consists of five parts. Please note that in parts II through V, you can skip one question.

Part I: Multiple Choice (mixed new and review questions) [50 points] (2 points each)

For each question, choose the best answer

- When an atom makes a transition from some level to another level, what normally occurs?
A) A photon is emitted, whether it goes up in energy or down in energy
B) A photon is absorbed, whether it goes up in energy or down in energy
C) A photon is absorbed if it goes up in energy, and emitted if it goes down in energy
D) A photon is emitted if it goes up in energy, and absorbed if it does down in energy
E) None of the above
- The energy of a photon with wavelength λ and frequency f is given by
A) hf B) $h\lambda$ C) h/f D) f/h E) $f\lambda$
- Below are the masses of the isotopes of Tungsten in u. Which one is ^{183}W ?
A) 179.9467 B) 181.9482 C) 182.9502 D) 183.9509 E) 185.9544
- For a perfect black body distribution, what determines the wavelength with the most power in it?
A) It is directly proportional to the temperature
B) It is inversely proportional to the temperature
C) It is directly proportional to the energy density
D) It is directly proportional to the frequency with the most power
E) None of the above
- If you calculate the expectation value of the position, $\langle x \rangle$, what does the result tell you?
A) Where the particle will definitely be found
B) The average location for the particle if you repeat the measurement many times
C) The most likely place to find the particle
D) The least likely place to find the particle
E) The range of x -values where one might find the particle
- When we say that an electron is in a 4d subshell of hydrogen, what do the 4 and d tell us?
A) Energy level and total angular momentum
B) Energy level and angular momentum around z -axis
C) Energy level and spin
D) Total angular momentum and angular momentum around z -axis
E) Total angular momentum and spin

7. An object made of three anti-quarks would be considered a(n)
 A) Meson B) Baryon C) Lepton D) Neutrino E) Anti-baryon
8. Which of the following is always conserved in special relativity?
 A) Energy (only)
 B) Momentum (only)
 C) Mass (only)
 D) Energy and momentum, but not mass
 E) Energy, momentum, and mass
9. Which of the following might be the diameter of the nucleus of an atom?
 A) 3×10^{-9} m B) 3×10^{-11} m C) 3×10^{-13} m D) 3×10^{-15} m E) 3×10^{-17} m
10. For a very heavy stable (or nearly) stable nucleus, what fraction of the nucleons would one expect to be protons?
 A) 30% B) 40% C) 50% D) 60% E) 70%
11. White dwarfs are held up by what force?
 A) Ideal gas pressure
 B) Radiation pressure
 C) Electron degeneracy pressure
 D) Neutron degeneracy pressure
 E) Microscopic but very strong invisible gnomes
12. If (Z, A) is the charge and mass of a nucleus, what does it change to if we undergo β^+ decay?
 A) $(Z, A+1)$ B) $(Z, A-1)$ C) $(Z-1, A)$ D) $(Z+1, A)$ E) (Z, A) unchanged
13. According to one observer, a clock moving at high velocity compared to her will run
 A) Slower
 B) Faster
 C) The same speed
 D) Slower if it is the observer that is moving; faster if it is the clock that is moving
 E) Faster if it is the observer that is moving; slower if it is the clock that is moving
14. When electrons shift levels in ^2H , they have slightly different energies than in ^1H . Why?
 A) The electrons in ^2H have a slightly different mass
 B) The extra neutron in ^2H has a little bit of charge (normally ignored) that causes this shift
 C) Though the extra neutron is neutral, it does have magnetic interactions with the electron, which cause this shift
 D) The nucleus is a slightly different size, which changes the interaction with the electron
 E) As the electron orbits, the nucleus moves a little too, so you have to use the reduced mass, which depends (slightly) on the mass of the nucleus

15. In general relativity, what causes curvature?
- Energy density (only)
 - Pressure (only)
 - Mass (only)
 - The stress-energy tensor, which include things like energy density and pressure
 - Light
16. The Sun is what kind of star?
- Pre-main sequence
 - Main sequence
 - Red giant
 - White dwarf
 - Neutron star
17. Light coming from the surface of a high gravity object like a neutron star will be changed in what way, according to general relativity?
- It will be shifted to a shorter wavelength
 - It will be shifted to a longer wavelength
 - It will remain as light with no change in wavelength
 - It will be converted to neutrinos
 - It will fail to escape, because it can't cross the event horizon
18. The reason that the strong force that holds the nucleus together is short range is because
- The particles that carry it (pions) are massive rather than massless
 - Nucleons interact via direct contact, not by exchanging particles
 - A cloud of screening particles blocks out interactions at long distances
 - The total strong charge of a nucleus is zero, so there's no attraction far away
 - None of these is correct
19. In special relativity, there is a fourth component of momentum, which is
- Time
 - Mass
 - Distance
 - Angular Momentum
 - Energy
20. Which rule is true about fermions in particle physics interactions?
- The total number of fermions is always unchanged
 - The number of fermions on each side of the interaction must be odd
 - The number of fermions on each side of the interaction must be even
 - The total number of fermions (left plus right) must be odd
 - The total number of fermions (left plus right) must be even
21. What do all main sequence stars have in common?
- They all have about the same surface temperature
 - They all have about the same mass
 - They all have about the same radius
 - They all have about the same luminosity
 - They are all converting hydrogen to helium at their centers
22. For the infinite square well with allowed region $0 < x < a$, which of the following formulas makes sense, given that the boundary is at 0?
- $\psi = e^{ax}$
 - $\psi = e^{-ax}$
 - $\psi = \cos(kx)$
 - $\psi = \sin(kx)$
 - $\psi = e^{-Ax^2}$

23. Which of the following formulas is still true in special relativity?
A) $F = ma$ B) $E = \frac{1}{2}mv^2$ C) $W = Fd$ D) $p = mv$ E) None of these is correct
24. Which sorts of objects have produced gravity waves that have been measured?
A) Rotating but otherwise stationary black holes
B) Merging black holes
C) Exploding stars
D) Particles colliding at high speeds in particle colliders
E) Gravity waves have not yet been detected
25. Which of the following particles has strong interactions?
A) Electron B) Photon C) Neutrino D) Proton E) None of these

Part II: Short answer (review material) [20 points] (10 points each)

Choose **two** of the following three questions and give a short answer (1-3 sentences)

26. According to special relativity, how do moving objects compare in length to objects at rest?
Can this be used to determine which of two observers is actually moving, and which is at rest?
27. What assumptions in the Bohr model of the atom allow one to explain why different atoms absorb or emit light at only specific wavelengths. Equations are not necessary.
28. Explain qualitatively what happens, according to quantum mechanics, when a particle of energy E impacts a barrier of finite width with a potential energy $V_0 > E$.

Part III: Short answer (new material) [30 points] (10 points each)

Choose **three** of the following four questions and give a short answer (1-3 sentences) .

29. Explain what a half-life is. In particular, if we started with 100,000 atoms of some isotope, how many would be left after two half-lives. Explain how you can find out the decay rate λ given the half-life.
30. Name at least five quarks in the standard model. Name at least four of the force carrying particles in the standard model. Finally, name the particle in the standard model which is responsible for all masses.
31. The main sequence of stars forms a diagonal band on the Hertzsprung-Russel diagram. How do the stars in the upper left compare with those in the lower right in terms of temperature, luminosity, and mass compare with the lower right. Is the Sun on the main sequence, and where?
32. “Matter tells space how to curve, and space tells matter how to move.” Explain this statement. Your answer should probably include words like “stress-energy tensor” and “geodesic.”

Part IV: Calculation (review material) [40 points] (20 points each)

Choose **two** of the following three questions and perform the indicated calculations

33. Protons have a mass of $938 \text{ MeV}/c^2$. A particular proton, however, has an energy of $E = 1437 \text{ MeV}$.
- (a) What is the velocity, both as a fraction of c , and in m/s?
 - (b) Assume a proton at rest is normally a sphere of diameter exactly 1.75 fm . If the proton is moving to the right, what shape will it have, and what will its diameter be in each of the three directions?
 - (c) The proton goes at this speed for $4.80 \text{ ns} = 4.80 \times 10^{-9} \text{ s}$, as measured in its own frame. How far does it go, as measured by us?

34. A particle has wave function $\psi(x) = \begin{cases} N \sin(3\pi x/a) & 0 \leq x \leq a, \\ 0 & \text{otherwise,} \end{cases}$

where N is an unknown normalization constant. Some possibly useful integrals are below.

- (a) What is the value of the constant N ?
- (b) At what place(s) in the allowed region will the particle definitely not be?
- (c) What is the most probable place(s) to find the particle?
- (d) What is the probability density that the particle is at the point $x = \frac{1}{9}a$?

Possibly useful integrals: $\int_0^a \sin(n\pi x/a) dx = \frac{a}{\pi n} [1 - (-1)^n]$, $\int_0^a \sin^2(n\pi x/a) dx = \frac{1}{2}a$

35. A helium atom ($Z = 2$) with one electron in it has an energy of $E = -3.40 \text{ eV}$.
- (a) What is the principle quantum number n for this electron?
 - (b) What are the possible values that the orbital angular momentum squared L^2 , the z -component L_z , the total spin square S^2 , and the z -component of the spin S_z take on, based on the value of n you found in part (a)? In each case, your answer might be a single value or a list of values.
 - (c) The angular momentum around the z axis is observed to be $L_z = -2\hbar$. Based on this additional piece of information, what would you change about your answers to part (b), if anything?

Part V: Calculation (new material): [60 points]

Choose **three** of the following four questions and perform the calculations (20 points each)

36. The isotope ^{238}U decays via α decay. This is followed by β^- decay, β^+ decay, and α decay. For this problem, you may fill in your answers directly on the table, if you want. A table of isotopes appears on the last page of this test.

| Nucleus | A | Z | N | Decay | Q (MeV) |
|------------------|-----|-----|-----|-----------|-----------|
| ^{238}U | | | | α | |
| | | | | β^- | |
| | | | | β^+ | |
| | | | | α | |
| | | | | | |

- (a) Write the values of Z and A for the initial nucleus, and for the resulting nucleus after each of the four steps. Note that in each step, the daughter nucleus becomes the parent for the next step. Then fill in the type of nucleus (^{238}U , etc.)
- (b) For each parent nucleus, calculate the number of neutrons (N). Based on what you know about stability of nuclei vs. even or odd, which of these nuclei do you think is least stable and decays most rapidly?
- (c) What is the Q -value for each step?

37. The Ω^- is a baryon with mass $1672 \text{ MeV}/c^2$ and strangeness -3 . Its most common decay is $\Omega^- \rightarrow \Lambda^0 K^-$, where Λ^0 and K^- each have strangeness -1 , the Λ^0 has a mass of $1116 \text{ MeV}/c^2$, and the K^- is a meson.

- (a) Is the Λ^0 a baryon, an anti-baryon, or a meson?
- (b) What is the charge of the K^- ? What is its maximum possible mass?
- (c) Is this interaction, strong, weak, or electromagnetic?
- (d) Only up, down, and strange quarks (or their anti-quarks) are found in these three particles. What is the quark composition of each of these particles?

38. The star Betelgeuse has a luminosity of approximately $L = 120,000L_\odot$ ($L_\odot = 3.828 \times 10^{26} \text{ W}$) and a surface temperature of $T = 3590 \text{ K}$ ($T_\odot = 5772 \text{ K}$).

- (a) What is the radius of Betelgeuse, both compared to the Sun's radius $R_\odot = 6.957 \times 10^5 \text{ km}$, and as compared to the Sun-Earth distance ($\text{AU} = 1.496 \times 10^8 \text{ km}$)?
- (b) What is the radiation pressure near the surface, assuming it is a black body, in N/m^2 ?
- (c) Betelgeuse is about 222 pc away ($1 \text{ pc} = 3.086 \times 10^{16} \text{ m}$). Assuming Betelgeuse radiates power equally in all directions, what is the flux (W/m^2) of radiation here?

39. A researcher is studying the black hole at the center of a galaxy with unknown mass. He spends two days, as measured by him, at a distance of 982 AU ($\text{AU} = 1.496 \times 10^{11} \text{ m}$) from the center of the black hole.

- (a) When he rejoins his companions far from the black hole, he discovers that his clock is off from other clocks by exactly one hour. Will it be "slow" (i.e., behind them by an hour) or "fast" (i.e., ahead of them by an hour).
- (b) What is the Schwarzschild radius for this black hole, in AU ?
- (c) What is the mass of the black hole, in solar masses ($M_\odot = 1.989 \times 10^{30} \text{ kg}$)?

Equations

$$\begin{aligned}
 h &= 6.626 \times 10^{-34} \text{ J} \cdot \text{s} = 4.136 \times 10^{-15} \text{ eV} \cdot \text{s} & u &= 931.5 \text{ MeV} / c^2 \\
 \hbar &= 1.055 \times 10^{-34} \text{ J} \cdot \text{s} = 6.582 \times 10^{-16} \text{ eV} \cdot \text{s} & u &= 1.661 \times 10^{-27} \text{ kg} \\
 \text{Constants: } G &= 6.673 \times 10^{-11} \text{ m}^3 / \text{kg} / \text{s}^2 & 2m_e c^2 &= 1.022 \text{ MeV} \\
 N_A &= 6.022 \times 10^{23} & M_{\text{He}} &= 4.0026 \text{ u} \\
 k_B &= 1.381 \times 10^{-23} \text{ J/K} = 8.671 \times 10^{-5} \text{ eV/K}
 \end{aligned}$$

$$\text{Hydrogen-Like atoms: } E_n = -\frac{(13.6 \text{ eV}) Z^2}{n^2}$$

$$\text{Radiation Pressure: } P_r = \frac{\pi^2 (k_B T)^4}{45 (\hbar c)^3} \quad \text{Stellar Luminosity: } \frac{L}{L_\odot} = \left(\frac{R}{R_\odot} \right)^2 \left(\frac{T}{T_\odot} \right)^4$$

$$\text{Gravitational time dilation: } \tau = t \sqrt{1 - \frac{2GM}{c^2 r}} \quad \text{Schwarzschild radius: } R_s = \frac{2GM}{c^2}$$

Isotope Masses

| <u>Z</u> | <u>Element</u> | <u>Sym</u> | <u>Mass#</u> | <u>Atomic Mass</u> |
|----------|----------------|------------|--------------|--------------------|
| 90 | Thorium | Th | 227 | 227.0277 |
| | | | 228 | 228.0287 |
| | | | 229 | 229.0318 |
| | | | 230 | 230.0331 |
| | | | 231 | 231.0363 |
| | | | 232 | 232.0381 |
| | | | 234 | 234.0436 |
| 91 | Protactinium | Pa | 231 | 231.0359 |
| | | | 234 | 234.0433 |
| 92 | Uranium | U | 231 | 231.0363 |
| | | | 232 | 232.0371 |
| | | | 233 | 233.0396 |
| | | | 234 | 234.0409 |
| | | | 235 | 235.0439 |
| | | | 236 | 236.0456 |
| | | | 238 | 238.0508 |
| 93 | Neptunium | Po | 235 | 235.0441 |
| | | | 236 | 236.0466 |
| | | | 237 | 237.0482 |