December 11, 2019 Name ________________

Final Exam

This test consists of five parts. Please note that in parts II through V, you can skip one question.

Part I: Multiple Choice (mixed new and review questions) [50 points] (2 points each)
For each question, choose the best answer

1. According to general relativity, if you go inside the Schwarzschild radius for a black hole, nothing can escape, not even light. But the same formula applies for the metric around the Earth. Why don't we notice this effect?
   A) There is light from the Sun in our area, which makes the Earth not black
   B) The Schwarzschild radius is relevant only for high mass objects, and the Earth is too low mass for there to be such an effect
   C) The Schwarzschild radius is deep within the Earth; if we could somehow reach this, then we would observe this effect
   D) The Schwarzschild metric applies only outside the Earth, and the Schwarzschild radius is inside the Earth, where the formula doesn't apply
   E) It does apply, but because we are inside the Earth's Schwarzschild radius, we can see the light

2. The particle that exchanges the force between the nucleons in a nucleus are called
   A) Neutrons B) Neutrinos C) Photons D) W-bosons E) Pions

3. If an electron has $l = 3$, what would be the value of the angular momentum squared $\vec{L}^2$?
   A) $3\hbar^2$ B) $6\hbar^2$ C) $9\hbar^2$ D) $12\hbar^2$ E) $18\hbar^2$

4. Which of the following is <u>not</u> a prediction of special relativity
   A) Moving clocks slow down
   B) By measuring the relative speed of two clocks, you can determine which one is actually moving
   C) There are no rigid objects
   D) No object can move faster than light
   E) Massless objects always move at the speed of light

5. The fundamental reason large nuclei tend to α-decay is because
   A) There are a lot of protons repelling each other
   B) The α-particle isn't affected much by the strong force
   C) The nuclear force becomes repulsive at short range, forcing the α-particle out
   D) There are so many electrons that they electrostatically pull the α-particle out
   E) The α-particle is so large it can't fit inside the nucleus

6. According to the uncertainty principle, there is a lower limit on the ________ of the uncertainties of the position Δx and the momentum Δp.
   A) Sum B) Difference C) Average D) Ratio E) Product

7. According to general relativity, what effect does the *rotation* of the Earth have on objects near it?
   A) It pulls spacetime around with it in an effect called *frame dragging*
   B) It is the reason that orbits precess, rather than moving in constant elliptical orbits
   C) It causes gravitational radiation that makes the orbits slowly decay
   D) It is the *cause* of gravity; if the Earth weren't rotating, it would have no gravity
   E) None; as long as the Earth is spherical, it doesn't matter if it is rotating

8. A nucleus with four neutrons and three protons would be (Li: Z=3, Be: Z=4, N: Z=7)
   A) ^{4}Li B) ^{7}Li C) ^{3}Be D) ^{7}Be E) ^{3}N

9. Which of the following properties is *not* currently explained by the standard model?
   A) How the electron gets its mass
   B) Why the Z and W bosons have mass
   C) How neutrinos get their mass
   D) Why quarks are confined inside baryons and mesons
   E) How action at a distance can occur in electromagnetism

10. Which of the following formulas gives the energy of one photon?
   A) $\hbar\omega$ B) h/p C) p/h D) $\hbar f$ E) $p^2/(2m)$

11. Which particles do <u>not</u> have wave properties, according to quantum mechanics?
   A) Photons B) Electrons C) Atoms D) Molecules E) All particles have wave properties

12. Which of the following is <u>false</u> about gravitational waves
   A) They have not yet been directly experimentally observed
   B) Direct detection of these waves involves measuring distances in perpendicular directions to incredible precision
   C) They have been indirectly detected by studying neutron star pairs (pulsars) orbiting each other
   D) They are produced by accelerating masses
   E) They represent real distortions of distance, not just forces

13. The name of the tensor that is proportional to the stress-energy tensor in general relativity is the _______ tensor
   A) Riemann B) Ricci C) Metric D) Einstein E) Carlson

14. Which of the following is the correct formula for the momentum operator in one dimension?
   A) $\frac{\hbar}{i}\frac{\partial}{\partial x}$ B) $m\frac{dx}{dt}$ C) $-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}$ D) $\frac{h}{\lambda}$ E) None of these

15. The most penetrating and dangerous type of radiation comes from which nuclear decay?
A) α B) β^+ C) β^- D) γ E) electron capture

16. Which of the following is not an assumption of the Bohr model of hydrogen?
A) The angular momentum of the electron is always a multiple of $\hbar$
B) The electron orbits the nucleus in circular orbits
C) When the electron falls from one level to another, a single photon carries off the energy
D) The electron's position is represented not by a specific value, but by a wave function ψ
E) Actually, all of these are assumptions of the Bohr model

17. The gluon is believed to be the particle responsible for which force?
A) Weak B) Gravity C) Electric D) Magnetic E) Strong

18. If I am traveling to the right at speed 0.6*c* and I send a light beam to the left (in vacuum), how fast will it move according to a stationary observer?
A) 0.4*c* B) 0.6*c* C) *c* D) 1.6*c* E) None of these

19. Which of the following wave functions, if valid between $-\infty$ and ∞, would be normalizable?
A) $\psi = e^{ax}$ B) $\psi = e^{-ax}$ C) $\psi = Bx^2$ D) $\psi = e^{+Ax^2}$ E) $\psi = e^{-Ax^2}$

20. Quarks inside a Δ^{++} baryon have in addition to their flavor (up) and charge (+2/3), another property that makes them not identical called
A) Isospin B) Color C) Mass D) Parity E) Hypercharge

21. The expectation value of the Hamiltonian will tell you the average value of the ________ if you were to measure a particle
A) Position B) Mass C) Energy D) Time E) Momentum

22. Under what circumstances can you tell which of two events *A* and *B* occurred first according to all observers?
A) As long as they are not at the same time as viewed by any observer
B) When they are spacelike separated
C) When they are timelike separated
D) Only if they occur at the same place
E) Never; it is always ambiguous

23. Which of the following particles is not a fermion?
A) Electron B) Up quark C) Photon D) Neutrino 1 E) Bottom quark

24. A cube of dimension 1×1×1 is moving in the *x*-direction at speed such that $\gamma = 2$. What would be the dimensions of the cube as measured by someone not moving with it?
A) ½×½ ×½ B) 1×½ ×½ C) ½×1×1 D) 2×2×2 E) 2×1×1

25. Which of the following is approximately the size of an atom?
A) 10^{-8} m B) 10^{-10} m C) 10^{-12} m D) 10^{-14} m E) 10^{-16} m

Part II: Short answer (review material) [20 points] (10 points each)
Choose **two** of the following three questions and give a short answer (1-3 sentences)

26. In relativity, nothing moves faster than light, but surely you could at least *signal* faster than light simply by, say, pushing on a rigid pole and detecting it at some distant place. Comment.

27. Explain what Rutherford was able to measure and to deduce by scattering α-particles off of gold atoms.

28. If you have a potential for a particle that changes at a boundary, you often have to calculate the wave function on the two sides of the potential, and then join them together. What conditions do you place on how the wave functions on the two sides are joined?

Part III: Short answer (new material) [30 points] (10 points each)

Choose **three** of the following four questions and give a short answer (1-3 sentences) .

29. Explain, if you know the mass, charge, and spin of a particle, you would find the corresponding numbers for its anti-particle. Do all particles have anti-particles? Are the anti-particles always different from the particles?

30. What sort of nuclei are likely to undergo (a) β^- decay, (b) β^+ decay or electron capture, and (c) α decay.

31. Name at least three conservation laws that are respected by all particle physics forces, and name one that is respected by strong and electromagnetic forces, but not weak forces.

32. According to Newton, two neutron stars would orbit each other in an elliptical pattern, and will do so forever. Explain at least two ways in which Einstein's theory of gravity disagrees with this prediction.

Part IV: Calculation (review material) [40 points] (20 points each)

Choose **two** of the following three questions and perform the indicated calculations

33. In principle (probably not in practice), one way to make the J/ψ particle is to collide a neutron and an anti-neutron head on, each of them moving at $v = 2.38\times10^8$ m/s.
    (a) What is the Lorentz factor γ for the neutron and anti-neutron?
    (b) The neutron has a mass of $m = 940\ \text{MeV}/c^2$. What is the mass of the anti-neutron? What is the energy of each of these particles? What is the mass of the J/ψ particle?
    (c) A neutron has a mean lifetime in its own frame of 882 s. What is the mean lifetime of these moving neutrons, and how far can they go before they decay?

34. A particle has wave function $\psi(x) = \begin{cases} N\sqrt{a^2x - x^3} & 0 \le x \le a\,, \\ 0 & \text{otherwise,} \end{cases}$
    where N is an unknown normalization constant.
    (a) What is the value of the constant N?
    (b) What is the most probable place(s) to find the particle?
    (c) What is the probability that the particle is in the region $0 < x < \frac{1}{2}a$?

35. Tin is illuminated with a frequency of $f = 1.47\times10^{15}\ \text{s}^{-1}$. The tin is found to emit electrons which can overcome a potential of up to $V_{\max} = 1.66\ \text{V}$.
    (a) What is the work function for tin?
    (b) Suppose the light was changed to a wavelength of $\lambda = 153\ \text{nm}$. What would be the maximum voltage that could be overcome?
    (c) For what frequency would the electrons be able to overcome a potential of $V_{\max} = 2.32\ \text{V}$?
    (d) What is the longest wavelength that can remove an electron from tin?

Part V: Calculation (new material): [60 points]
Choose **three** of the following four questions and perform the calculations (20 points each)

36. One of the rarer isotopes of potassium is ^{40}K. A certain sample of ^{40}K has a mass of 4.02×10^{-3} g, and undergoes approximately 1038 decays every second
    (a) What is the approximate mass in u of one atom of ^{40}K? How many atoms of ^{40}K do we have?
    (b) What is the decay rate λ in s^{-1}? What is the half-life in y? ($y = 3.156\times10^{7}$ s)
    (c) The Earth is 4.56×10^{9} y old. Assuming this sample has been around that whole time, how many ^{40}K atoms were there to begin with? What was their total mass in g?

37. On the nest page is a list of isotopes and their masses. ^{108}Ag is a nucleus that might decay by one of the modes listed at right. You may use the table at right to summarize your answers.
    (a) For each of these, what is the daughter isotope?
    (b) What is the Q-value for each decay?
    (c) Which of the modes can actually occur?

mode	Daughter	Q (MeV)	?
α			
electron capture			
β^+			
β^-			

38. There is a particle Ξ^{*-} which decays by strong interactions as follows: $\Xi^{*-} \rightarrow \Sigma^- + X$. The Ξ^{*-} and Σ^- are both baryons, and the other particles in the table at right are all mesons. The spin and strangeness of the other particles are listed at right, and the charges are implied by their names.
    (a) What is the charge and strangeness of the X particle?
    (b) Is it a baryon, anti-baryon, or a meson?
    (c) What, if anything, can you conclude about the mass of the X?
    (d) Could the X be any of the particles in the table given? Could it be any of their anti-particles?

All masses in MeV/c^2

Name	Mass	Spin	Strange
Ξ^{*-}	1820	3/2	–2
Σ^-	1197	1/2	–1
π^0	135	0	0
K^-	494	0	–1
K^0	498	0	+1
$\bar{K}^{*0}$	892	1	–1

39. A neutron star of radius $R = 11.5$ km is being studied by a researcher. She is far from the neutron star, and discovers that a spectral line normally at 582 nm is observed, by her, at a wavelength of 762 nm.
    (a) What is the mass of the neutron star, in solar masses ($M_\odot = 1.989\times10^{30}$ kg)?
    (b) If she were to travel to a distance $R = 27.00$ km from the center of the neutron star, and spent 5.00 days doing research there (as measured by her), how much time would pass according to an observer far from the neutron star?
    (c) To what radius would this neutron star have to shrink to become a black hole?

Equations

Constants:

$$h = 6.626\times10^{-34}\ \text{J}\cdot\text{s} = 4.136\times10^{-15}\ \text{eV}\cdot\text{s}$$
$$\hbar = 1.055\times10^{-34}\ \text{J}\cdot\text{s} = 6.582\times10^{-16}\ \text{eV}\cdot\text{s}$$
$$N_A = 6.022\times10^{23}$$
$$G = 6.674\times10^{-11}\ \text{m}^3\text{kg}^{-1}\text{s}^{-2}$$

$$\text{u} = 931.5\ \text{MeV}/c^2 = 1.661\times10^{-27}\,\text{kg}$$
$$2m_e c^2 = 1.022\ \text{MeV}$$
$$M_{\text{He}} = 4.0026\ \text{u}$$

Photoelectric effect: $eV_{\max} = hf - \phi$

Gravitational time dilation and red shift: $\tau = t\sqrt{1-\frac{2GM}{c^2 r}}$, $\lambda = \frac{\lambda_0}{\sqrt{1-\frac{2GM}{c^2 r}}}$

Schwarzschild radius: $R_S = \frac{2GM}{c^2}$

Isotope Masses

Z	Element	Symbol	*A*	Atomic Mass
45	Rhodium	Rh	102	101.906794
			103	102.905502
			104	103.906654
46	Palladium	Pd	102	101.905616
			104	103.904033
			105	104.905082
			106	105.903481
			107	106.905126
			108	107.903893
			110	109.905158
47	Silver	Ag	107	106.905091
			108	107.905953
			109	108.904754
			110	109.906110
48	Cadmium	Cd	106	105.906457
			108	107.904183
			109	108.904984
			110	109.903004
			111	110.904182
			112	111.902760
			113	112.904401
			114	113.903359
			116	115.904755

Z	Element	Symbol	*A*	Atomic Mass
49	Indium	In	113	112.904060
			114	113.904916
			115	114.903876
			116	115.905258