Final Name _____
December 11, 2020

This test consists of three parts. Please note that in parts II and III, you can skip one question of those offered.

Part I: Multiple Choice [50 points]
For each question, choose the best answer (2 points each). Your test will have had these questions in a random order.

2. Which of the following is true in the proximity of a large mass, according to general relativity?
   A) The circumference of a circle of radius r is still $2\pi r$
   B) The gravitational acceleration of an object will depend on its mass
   C) Time slows down
   D) Time speeds up
   E) Spacetime is still perfectly flat

3. Under what circumstances is it wise to solve a problem using spherical coordinates (r,θ,ϕ) instead of Cartesian coordinates (x,y,z)
   A) When the potential is spherically symmetric, so it depends only on the radius r
   B) When you are looking for solutions with no angular momentum, l=0 .
   C) When it is unclear which axes to pick for x, y, and z.
   D) When we need the wave function to fall off at infinity in all directions
   E) When solving problems with no potential, like for plane wave solutions

4. Which of the following methods would be a good way to measure a gravity wave?
   A) Measure how the gravitational acceleration of objects near the Earth changes as it passes
   B) Measure distances with high accuracy, such as by using a very long interferometer
   C) Measuring how the mass of an object changes as it passes
   D) Carefully monitoring changes in the orbit of the Earth
   E) Checking for small electromagnetic waves generated by the gravity wave

5. In the most stable nuclei, the number of protons tends to be _____ and the number of neutrons tends to be _____ .
   A) Odd, odd B) Even, even C) Even, odd D) Odd, even E) Prime, composite

6. The particles that are exchanged inside the nucleus that hold the protons and neutrons together are called
   A) pions B) photons C) gravitons D) Z-bosons E) Nucleons

7. Which of the following is one of the assumptions made by the Bohr model?
   A) The electron does not have a definite position, but rather is described by a wave function
   B) All particles must satisfy the uncertainty relationship
   C) The number of waves that fit around the circular orbit must be an integer
   D) When an electron goes from one level to another in an atom, a single photon is emitted or absorbed
   E) The positive charge of an atom is spread throughout it, while electrons are stuck in this background; the "plum pudding" model

8. Under what conditions will two observers inevitably agree which of two events A and B came first?
   A) Only if they are timelike separated
   B) Only if they are spacelike separated
   C) They will always agree
   D) They will never be guaranteed to agree
   E) Only if both observers were present at both events

9. According to special relativity, all vectors should have four components. What is the fourth component of the vector corresponding to momentum?
   A) Mass B) Velocity C) Time D) Distance E) Energy

10. A ^{7}Be atom will decay via electron capture, but an isolated nucleus of ^{7}Be is stable. How is this possible?
   A) The negatively charged electrons in the atom pull out the decay products
   B) The electrons pull on the nucleus, making it larger and less stable
   C) The electrons can annihilate with the neutrinos that are produced, creating more energy
   D) Electron capture requires electrons that can be captured
   E) An atom without electrons will have a higher priority finding electrons rather than decaying

11. Which of the following is not generally conserved in relativity?
   A) Energy (only)
   B) Momentum (only)
   C) Mass (only)
   D) None of the above are conserved
   E) All of the above are conserved

12. Which event caused the universe to transform from being opaque to transparent?
   A) Neutron/proton freezeout
   B) Electron/positron annihilation
   C) The Grand Unified Theory (GUT) transition
   D) Recombination
   E) Primordial Nucleosynthesis

13. The physical velocity some object moves at corresponds to, when you think of it as a wave, is
    A) The phase velocity
    B) The group velocity
    C) The speed of light
    D) The node velocity
    E) The anti-node velocity

14. The universe is currently flat, or very close to flat. Which early universe event is conjectured to be the reason for this?
    A) The growth of density perturbations
    B) Primordial nucleosynthesis
    C) The transition from radiation domination to matter domination
    D) Recombination
    E) Inflation, a period of rapid growth

15. What information did Rutherford glean by scattering α−particles off of gold and aluminum foil?
    A) The density of matter inside atoms is very low
    B) Most of the deflection was caused by the electrons
    C) Electrons are negatively charged
    D) The positive charge and most of the mass of an atom is spread throughout the atom
    E) The positive charge and most of the mass of an atom is concentrated in a tiny region of the atom

16. How many quantum states are there in a 3d subshell?
    A) 3 B) 4 C) 5 D) 10 E) 12

17. Suppose I have two objects each of mass m, and they are moving towards each other at high velocity. The effective mass of the system of two masses will be
    A) Equal to $2m$
    B) Greater than $2m$
    C) Less than $2m$, but definitely positive
    D) Less than $2m$; it could even be negative or zero
    E) Insufficient information

18. The minimum positive energy that an electromagnetic wave of frequency f or angular frequency ω can have, according to Planck, is given by
    A) hf B) $\frac{1}{2}hf$ C) $\frac{1}{2}\hbar\omega$ D) $\frac{\pi^2\hbar^2}{2ma^2}$
    E) There is no minimum energy; all positive energies are possible

19. What is the difference between a protostar (pre-main sequence) and a main sequence star?
    A) Protostars are mostly helium; main sequence are mostly hydrogen
    B) Protostars are mostly hydrogen; main sequence are mostly helium
    C) The core of protostars are not hot enough for fusion; main sequence stars are hot enough
    D) Protostars mostly transport energy via radiation; main sequence mostly by convection
    E) Protostars have high mass; main sequence stars are low mass

20. Suppose you have a solution to Schrodinger's time-independent equation, but unfortunately the normalization integral $\int |\psi(x)|^2 \, dx = 4$. What can you do to fix the problem?
    A) Multiply the energy by four
    B) Divide the wave function by four
    C) Divide the wave function by two
    D) Multiply the wave function by two
    E) Nothing; if it isn't normalized properly you have to start over

21. The most important source of pressure for a white dwarf star is
    A) Ideal gas pressure
    B) Radiation pressure
    C) Electrostatic repulsion
    D) Degeneracy pressure from the nuclei
    E) Degeneracy pressure from the electrons

22. Iron has Z = 26, and a certain isotope of iron has a mass of 53.93961 u. How many neutrons does it have?
    A) 26 B) 27 C) 28 D) 53 E) 54

23. According to Einstein's equations, the stress-energy tensor, which includes things like energy density and momentum density, is directly proportional to what?
    A) An object's acceleration
    B) An object's velocity
    C) An object's geodesic equation
    D) The Einstein tensor, a measure of curvature
    E) The rate of change of momentum of an object

24. If the wave function for a particle is given by $\psi(x)$, and the expectation value of an operator is given by $\langle O \rangle = \int \psi^*(x) O \psi(x) dx$, which of the following formulas would allow one to calculate the expectation value of the momentum?

A) $\int \frac{\hbar}{i} \frac{\partial}{\partial x} \psi(x) dx$

B) $\int \frac{\hbar}{i} \left[\frac{\partial}{\partial x} \psi^*(x) \right] \psi(x) dx$

C) $\int \frac{\hbar}{i} \frac{\partial}{\partial x} \left[\psi^*(x) \psi(x) \right] dx$

D) $\int \psi^*(x) \psi(x) \frac{\hbar}{i} \frac{\partial}{\partial x} dx$

E) $\int \psi^*(x) \frac{\hbar}{i} \frac{\partial}{\partial x} \psi(x) dx$

25. When we say the universe is homogeneous and isotropic, what do we mean?
A) It is the same at all places and in all directions
B) It is the same at all times
C) The same laws of physics apply everywhere, and all directions are equal
D) Most of the objects in the universe are approximately spherically symmetrical
E) The cosmic microwave background must have originated at all places and in all directions equally

26. The Hertzsprung-Russel diagram shows high temperatures where on the diagram?
A) Right
B) Left
C) Top
D) Bottom
E) The HR-diagram doesn't include temperature information

Part II: Short essay, review [20 points]

Choose two of the following three questions, and write a short essay (2-3 sentences). You may type both answers into the answer box at the end, or you may upload your answers as an image into the box. Each question is worth 10 points.

27A. Agree or disagree with this statement, and explain approximately what each observer would see: If two observers, A and B, are moving compared to each other, they can always figure out which one is actually moving by seeing which of their two clocks runs faster.

27B. Explain why it is that, when you shine a light on a piece of metal, electrons pop off (i) only if the wavelength is short enough, and (ii) immediately, even if the intensity of the light is low, rather than having to accumulate enough energy.

27C. Suppose that the angular momentum of an electron around the *z*-axis is given by $L_z = 4\hbar$. What is (i) the minimum possible value for the total angular momentum squared $\mathbf{L}^2$, and (ii) the possible values of the angular momentum S_z. (Feel free to write $\hbar$ as **hbar** as needed)

Part III: Short essay, new material [30 points]

Choose three of the following four questions, and write a short essay (2-3 sentences). You may type all three answers into the answer box at the end, or you may upload your answers as an image into the box. Each question is worth 10 points.

28A. For the most stable isotopes, what fraction of the nucleons will be protons? Explain how this varies for light vs. heavy nuclei. What will happen to a nucleus that has too many protons or too few neutrons?

28B. Main sequence stars like the Sun burn what isotope to create what isotope? Give at least two reasons why this process is not very fast.

28C. In the very early universe, there were approximately equal numbers of protons and neutrons. Explain qualitatively how this ratio changed, and what became of the vast majority of these protons and neutrons.

28D. According to Newton, Mercury's orbit around the Sun will be an ellipse. How, if it all, does this change according to Einstein's general theory of relativity?

Part IV: Calculation, Old Material: [40 points]
Choose two of the following three questions. Each question is worth 20 points. Type **only your answers** to each part into the essay box provided.

29. A woman who has just become pregnant has a mass of 69.0 kg. She gets aboard a spacecraft traveling at a velocity of 2.83×10^8 m/s .

a) What is her total energy E, as measured by us?
b) Assuming her baby takes the usual 0.75 year to gestate, as measured by her, how long will it be until the baby is born, according to us?
c) How far will she have traveled in this amount of time? 1 y = 3.156×10^7 s
d) The woman is 169 cm tall and measures 92 cm around her hips. What are the same two measurements, according to observers on the Earth? The rocket is traveling in the direction of her height.

30. The energy of Hydrogen-like atoms is given by $E_n = -\dfrac{13.6Z^2 \text{ eV}}{n^2}$. A boron atom ($Z$ =5) has a single electron in level n = 7.
a) Suppose it were to fall to the n = 5 state. Would it emit or absorb a photon to do this? What would be the corresponding energy of that photon?
b) What would be the wavelength of the corresponding photon from part (a)?
c) Suppose the atom, starting in level 7, absorbed a photon of wavelength 452 nm. What would be the resulting level n of the atom after absorption of the photon?

31. A particle in one dimension is in the region $0 \le x \le a$ and has wave function

$$\psi(x) = a^{-2}\sqrt{12x}\,(a-x) = a^{-2}\sqrt{12}\left(ax^{1/2} - x^{3/2}\right).$$

a) Where in the allowed region is it impossible for the particle to be?
b) Where in the allowed region is the particle most likely to be?
c) What is the probability that the particle is in the region $0 \le x \le \frac{1}{2}a$?

Part V: **Computation, New Material**

Choose three of the following four questions. Each question is worth 20 points. Type **only your answers to each part** into the essay box provided.

32. The isotope ^{230}Th is rare but does occur naturally. It has a half-life of $t_{1/2} = 75{,}400$ years. A small table of isotopes is given to help you with this problem.
    a) What is the decay λ in y^{-1}?
    b) If you started with a mole (6.022×10^{23}) of ^{230}Th atoms, how many years would it take to reduce this to just one atom? Since most rocks are much older than this, clearly the ^{230}Th could not have been there all along.
    c) The ^{230}Th comes from the α-decay of some other isotope. What isotope is creating ^{230}Th?
    d) Calculate the Q-value for the decay that produces ^{230}Th. The mass of ^{4}He is $m_{He} = 4.00260$ u.

Z	Sym.	*A*	Mass (u)
88	Ra	226	226.02540
		230	230.03706
90	Th	226	226.02490
		230	230.03313
		234	234.04359
92	U	230	230.03394
		234	234.04095

33. Listed at right are the names and some information about several stars. For each star, determine the missing information. The surface temperature of the sun is $T_\odot = 5778$ K.

Name	L ($L_\odot$)	R ($R_\odot$)	T (K)
Proxima Centauri		0.154	3,042
α-Centauri A	1.519		5,260
α-Centauri B	0.500	0.863	
Sirius A		1.711	9,940
Sirius B	0.056		25,000
Betelgeuse	126,000	764	

34. The current temperature of the universe is $T_0 = 2.7255$ K. This question is about when the temperature was the temperature of boiling water, $T = 373$ K. This is lower than the temperature at matter-radiation equality, 8000 K.
    a) What was the red-shift z at this time?
    b) What was the approximate age of the universe in My at this time? If you need it, $g_{eff} = 3.36$ at this time.
    c) The current density of the universe is around 0.251 atoms/m^3. What was the density of atoms at this time?
    d) What was the ideal gas pressure from these atoms at this time?

35. In the book *Dragon's Egg*, by Robert L. Forward, a group of creatures called *cheela* live on the surface of a neutron star of mass $0.510\,M_\odot$ (where $M_\odot = 1.989\times10^{30}$ kg), which has a radius of 20.3 km.
    a) Suppose the cheela on the surface send a signal to humans who are far from the neutron star at a wavelength of 483 nm. At what wavelength will the humans detect it?
    b) Most of the story takes place over the course of approximately 30.0 days, as measured by the humans. How much time will pass according to the cheela?
    c) Suppose that due to a natural disaster, the neutron star were to collapse to a black hole. What would be the resulting Schwarzschild radius of the black hole?