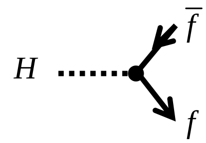
Solutions to Problems 11

1. **Find a formula for the rate of Higgs decay to leptons $H \to \ell^+\ell^-$ or any quark $H \to \bar{q}q$, for arbitrary Higgs mass M_H and arbitrary fermion mass *m*. Don't forget colors, when appropriate. Why did I leave out the neutrinos? For the actual Higgs mass (126 GeV), determine which rate(s) dominate, and estimate the total decay rate in MeV. We currently can resolve the particles produced with an accuracy of about 1 GeV. Are we close to directly measuring the width Γ for the Higgs? You may find problem 6.4 useful.**



The decay rate from problem 6.4 (as found in the online solutions) is

$$\Gamma\left(H \to f\bar{f}\right) = \frac{g^2}{8\pi M_H^2}\left(M_H^2 - 4m_f^2\right)^{3/2} .$$

However, comparison of the Feynman rules for this decay with those for the actual Higgs indicate that the coupling *g* should be replaced by m_f/v. Furthermore, there is a factor of three for colors. This gives us the decay rates:

$$\Gamma\left(H \to q\bar{q}\right) = \frac{3m_q^2}{8\pi v^2 M_H^2}\left(M_H^2 - 4m_q^2\right)^{3/2} , \quad \Gamma\left(H \to \ell^+\ell^-\right) = \frac{m_\ell^2}{8\pi v^2 M_H^2}\left(M_H^2 - 4m_\ell^2\right)^{3/2} .$$

The neutrinos are not included in the standard model calculation because the neutrinos are massless. Even if they have mass, it isn't obvious that the same couplings apply, and the masses are truly tiny, so they are irrelevant.

Because the coupling is proportional to the mass, we would expect the heaviest allowed particle to contribute the most. This would be the bottom quark. The tau and charm will also contribute significantly. Hence the dominant decay rates will be

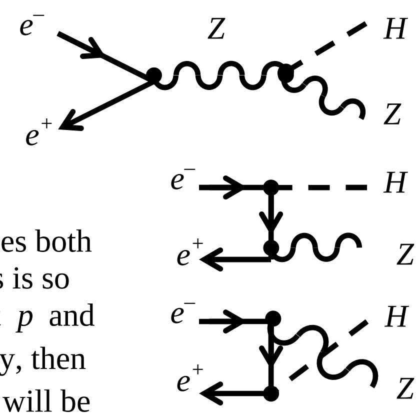
$$\Gamma\left(H \to b\bar{b}\right) = \frac{3m_b^2\left(M_H^2 - 4m_b^2\right)^{3/2}}{8\pi v^2 M_H^2} = \frac{3\left(4.13\text{ GeV}\right)^2\left[\left(126\text{ GeV}\right)^2 - 4\left(4.13\text{ GeV}\right)^2\right]^{3/2}}{8\pi\left(246\text{ GeV}\right)^2\left(126\text{ GeV}\right)^2}$$
$$= 0.00421\text{ GeV} = 4.21\text{ MeV} ,$$

$$\Gamma\left(H \to \tau^+\tau^-\right) = \frac{m_\tau^2\left(M_H^2 - 4m_\tau^2\right)^{3/2}}{8\pi v^2 M_H^2} = \frac{\left(1.777\text{ GeV}\right)^2\left[\left(126\text{ GeV}\right)^2 - 4\left(1.777\text{ GeV}\right)^2\right]^{3/2}}{8\pi\left(246\text{ GeV}\right)^2\left(126\text{ GeV}\right)^2}$$
$$= 0.00026\text{ GeV} = 0.26\text{ MeV} ,$$

$$\Gamma\left(H \to c\bar{c}\right) = \frac{3m_c^2\left(M_H^2 - 4m_c^2\right)^{3/2}}{8\pi v^2 M_H^2} = \frac{3\left(1.26\text{ GeV}\right)^2\left[\left(126\text{ GeV}\right)^2 - 4\left(1.26\text{ GeV}\right)^2\right]^{3/2}}{8\pi\left(246\text{ GeV}\right)^2\left(126\text{ GeV}\right)^2}$$
$$= 0.00039\text{ GeV} = 0.39\text{ MeV} .$$

This gives a total of about 4.86 MeV. Other tree level decays will be even smaller. This is some two orders of magnitude smaller than what we can currently directly measure, so it's pretty much impossible to deduce this rate experimentally.

3. In Fig. 11-5, I drew only one Feynman diagram for the process $e^+e^- \to Z^0 H$. Draw the other diagrams. Explain why they aren't relevant. Than write the Feynman amplitude for the relevant diagram. Simplify it insofar as possible. You don't have to do more.



The three diagrams are sketched at right. The two new ones both have the Higgs coupled to an electron, but since the electron mass is so tiny compared to the Z mass, it will be pretty negligible. If we let p and p' be the momenta of the initial electron and positron respectively, then the amplitude for the process (considering only the first diagram) will be given by

$$i\mathcal{M} = (ie)\frac{\bar{v}'\gamma^\mu\left(1-4\sin^2\theta_W-\gamma_5\right)u}{4\sin\theta_W\cos\theta_W}\cdot\frac{i}{(p+p')^2-M_Z^2}\left(-g_{\mu\nu}+\frac{(p_\mu+p'_\mu)(p_\nu+p'_\nu)}{M_Z^2}\right)\frac{2iM_Z^2 g^{\nu\alpha}}{v}\varepsilon_\alpha^*$$

Since we are essentially treating the electron as massless, we note that

$$\left[\bar{v}'\gamma^\mu\left(1-4\sin^2\theta_W-\gamma_5\right)u\right](p_\mu+p'_\mu) = \bar{v}'\left(\not{p}+\not{p}'\right)\left(1-4\sin^2\theta_W-\gamma_5\right)u$$
$$= \bar{v}'\not{p}\left(1-4\sin^2\theta_W-\gamma_5\right)u = \bar{v}'\left(1-4\sin^2\theta_W+\gamma_5\right)\not{p}u = 0.$$

Hence this horrendous expression can be simplified to

$$i\mathcal{M} = \frac{ieM_Z^2}{2v\sin\theta_W\cos\theta_W}\left[\bar{v}'\gamma^\alpha\left(1-4\sin^2\theta_W-\gamma_5\right)u\right]\varepsilon_\alpha^*\frac{1}{s-M_Z^2}$$

We can then use the identity $M_Z = ev/2\sin\theta_W\cos\theta_W$ to simplify this further to

$$i\mathcal{M} = \frac{iM_Z^3}{v^2\left(s-M_Z^2\right)}\left[\bar{v}'\gamma^\alpha\left(1-4\sin^2\theta_W-\gamma_5\right)u\right]\varepsilon_\alpha^*.$$

And that's about as simple as it's going to get.