Solutions to Problems 4a

1. **Three fermions would be described as $\left|a,b,c\right\rangle$, where we have combined the type, momentum and spin indices into a single letter. Show how all six orderings of the three states are related to each other.**

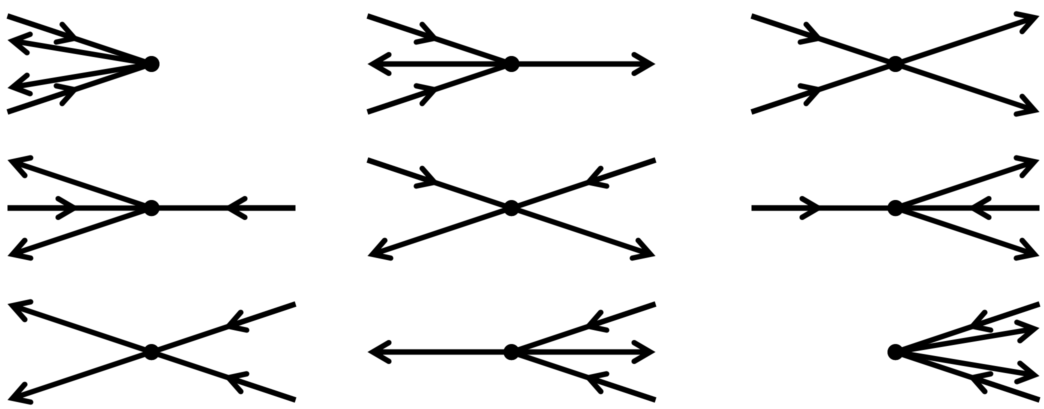
When you exchange any pair of states, you get a minus sign for fermions. It then isn't hard to see that

$$\left|a,b,c\right\rangle = \left|b,c,a\right\rangle = \left|c,a,b\right\rangle = -\left|b,a,c\right\rangle = -\left|c,b,a\right\rangle = \left|a,c,b\right\rangle.$$

3. **Write down all nine (eight plus the original one) matrix elements that correspond to the coupling λ_1 related by the anti-particle property. Also, draw all nine corresponding diagrams akin to Fig. 4-2 for this interaction.**

$$\left\langle 0\middle|\mathcal{H}\middle|\psi\psi\psi^*\psi^*\right\rangle = \left\langle \psi\middle|\mathcal{H}\middle|\psi\psi\psi^*\right\rangle = \left\langle \psi\psi\middle|\mathcal{H}\middle|\psi\psi\right\rangle =$$
$$\left\langle \psi^*\middle|\mathcal{H}\middle|\psi\psi^*\psi^*\right\rangle = \left\langle \psi\psi^*\middle|\mathcal{H}\middle|\psi\psi^*\right\rangle = \left\langle \psi\psi\psi^*\middle|\mathcal{H}\middle|\psi\right\rangle =$$
$$\left\langle \psi^*\psi^*\middle|\mathcal{H}\middle|\psi^*\psi^*\right\rangle = \left\langle \psi\psi^*\psi^*\middle|\mathcal{H}\middle|\psi^*\right\rangle = \left\langle \psi\psi\psi^*\psi^*\middle|\mathcal{H}\middle|0\right\rangle = \lambda_1 .$$

The corresponding pictures are below.



5. **Argue that $h, \lambda_1, \lambda_2,$ and λ_3 in eqs. (4.22) and (4.23) are constants, and they are all real.**

For *h*, it is pretty easy to see that $h = \left\langle 0\middle|\mathcal{H}\middle|\phi\phi\phi\right\rangle$ has dimensions of mass to the plus one, and therefore it contains at most only a constant term and a term linear in the momentum. But you can't make a Lorentz invariant quantity that is linear in the momentum, so it must be a constant. For $\lambda_1 = \left\langle 0\middle|\mathcal{H}\middle|\psi\psi\psi^*\psi^*\right\rangle$, $\lambda_2 = \left\langle 0\middle|\mathcal{H}\middle|\psi\psi^*\phi\phi\right\rangle$ and $\lambda_3 = \left\langle 0\middle|\mathcal{H}\middle|\phi\phi\phi\phi\right\rangle$, they have dimensions of mass to the zeroth, and hence they contain at most a constant term, and hence are constant.

We then use the Hermitian property to move all the particles to the other side, then use the anti-particle property to move them back, and we find

$$h^* = \langle \phi\phi\phi | \mathcal{H} | 0 \rangle = \langle 0 | \mathcal{H} | \phi\phi\phi \rangle = h,$$

$$\lambda_1^* = \langle \psi\psi\psi^*\psi^* | \mathcal{H} | 0 \rangle = \langle 0 | \mathcal{H} | \psi\psi\psi^*\psi^* \rangle = \lambda_1 ,$$

$$\lambda_2^* = \langle \psi\psi^*\phi\phi | \mathcal{H} | 0 \rangle = \langle 0 | \mathcal{H} | \psi\psi^*\phi\phi \rangle = \lambda_2 ,$$

$$\lambda_3^* = \langle \phi\phi\phi\phi | \mathcal{H} | 0 \rangle = \langle 0 | \mathcal{H} | \phi\phi\phi\phi \rangle = \lambda_3 .$$

Hence they are all real as well.