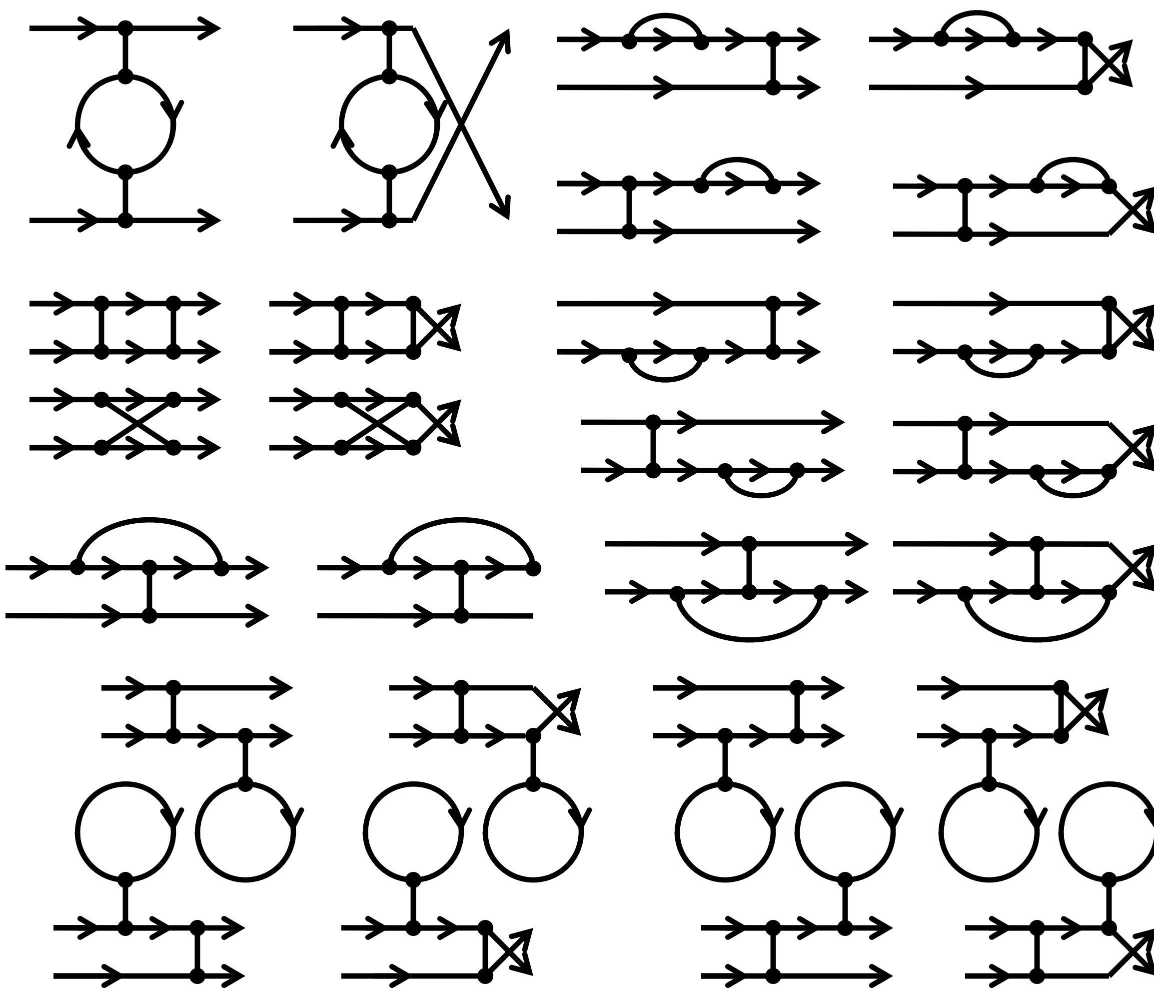
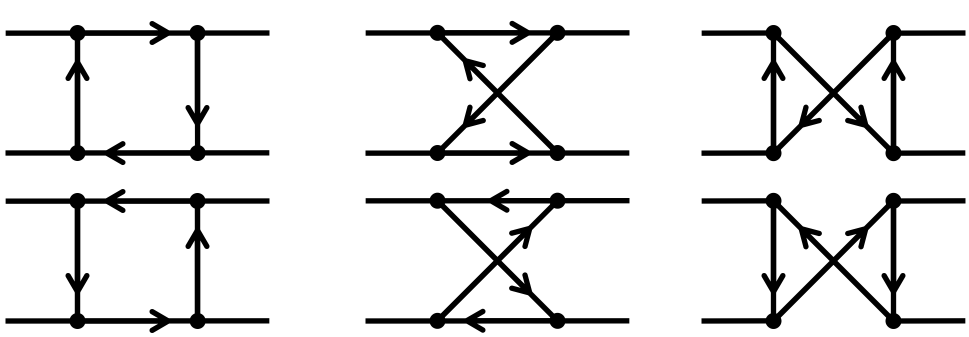
Solutions to Problems 5a

1. **Fig. 5-6 has a one loop diagram contributing to the scattering $\psi\psi \to \psi\psi$. Draw at least five more.**

The diagrams below are all the ones I could think of. The first one is from the text.

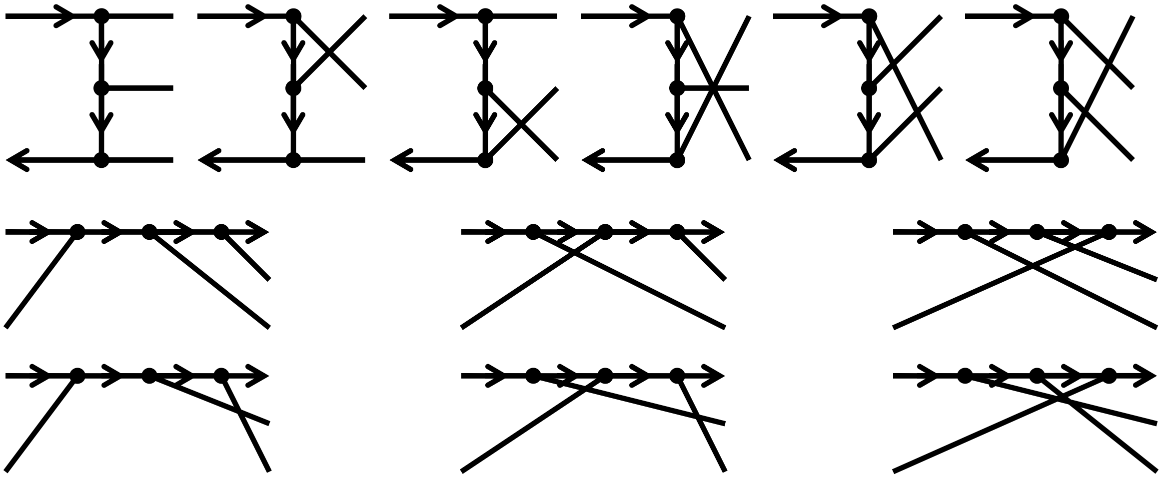


2. **In the restricted theory (*g* only), there is no tree-level diagram contributing to the process $\phi\phi \to \phi\phi$. Draw the one loop diagrams that *do* contribute to this process. There are six total.**



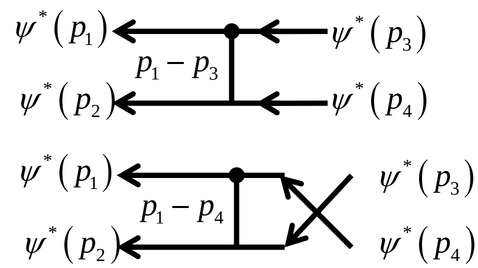
3. **In the restricted theory (*g* only), draw all six tree level diagrams contributing to $\psi\psi^* \to \phi\phi\phi$. Then draw all six diagrams contributing to $\psi\phi \to \psi\phi\phi$. You don't have to do anything else with them.**

The first row shows the six diagrams for $\psi\psi^* \to \phi\phi\phi$, and the next two rows for $\psi\phi \to \psi\phi\phi$.



5. **Calculate in the center of mass frame the cross-section for $\psi^*\psi^* \to \psi^*\psi^*$. Let p_1 and p_2 be the initial momenta, and p_3 and p_4 the final momenta. Once you get the Feynman invariant amplitude, I strongly recommend you compare with eq. (5.10) before finishing the problem.**

There are only two diagrams, sketched at right. The intermediate momenta are marked. Each of them has two factors of $-ig$ and one propagator with the ϕ particle of mass M. The Feynman invariant amplitude, therefore, is



$$i\mathcal{M} = \frac{(-ig)^2 i}{(p_1 - p_3)^2 - M^2} + \frac{(-ig)^2 i}{(p_1 - p_4)^2 - M^2}$$

Comparing to eq. (5.10), we see that this expression is identical, and all the particles involved have the same masses. We also have identical particles in the final state, so we can proceed directly to the final answer, which in the center of mass frame is

$$\sigma = \frac{g^4}{64\pi E^2}\left[\frac{\ln\left(1+4p^2/M^2\right)}{4M^2p^2+8p^4} + \frac{1}{M^4+4p^2M^2}\right]$$

$$= \frac{g^4}{16\pi s}\left[\frac{1}{M^2\left(s-4m^2\right)+\frac{1}{2}\left(s-4m^2\right)^2}\ln\left(1+\frac{s-4m^2}{M^2}\right) + \frac{1}{M^4+sM^2-4m^2M^2}\right].$$

These formulas were copied straight from p. 78.