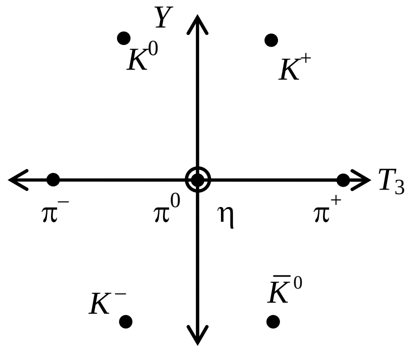
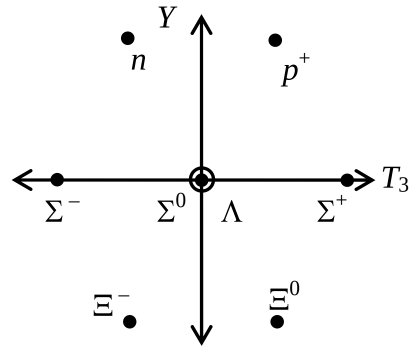
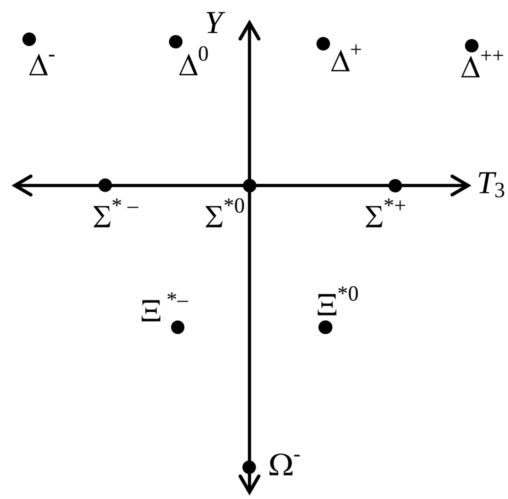
Solutions to Problems 8a

2. Since strong interactions preserve both Q and I_3, they must also preserve Y. Make a plot of I_3 vs. Y for the eight lightest spin-0 mesons, drawing a dot for each meson and labeling it by the particle name, and circling it if there are two at a particular point. Repeat for the eight lightest spin-1/2 baryons. Then do it for the nine lightest spin-3/2 baryons. Look up the Ω^- baryon from the particle data group and add it to the diagram. Comment on any similarities, differences, symmetries, etc. of the diagrams you end up with.







The bosons (upper left), spin-1/2 baryons (upper right) and spin 3/2 baryons (right) are sketched. The pattern for the first two are exactly identical. The shape is a regular hexagon with two particles in the center, except there is a vertical stretching by a factor of $2/\sqrt{3}$, for the first two cases, and an equilateral triangle stretched by the same factor in the final case.

4. Which matrix elements of the form $\langle \Lambda\pi | \mathcal{H} | \Sigma^* \rangle$ could be non-zero, based on charge conservation? Relate the three non-zero matrix elements using isospin symmetry, and make a prediction for the relative rates for the decays $\Gamma(\Sigma^* \to \Lambda\pi)$.

Since the Λ is neutral, the charge of the Σ^* must match the charge of the π. We can then relate the decay rates using

$$\begin{aligned}\langle \Lambda^0\pi^+ | \mathcal{H} | \Sigma^{*+} \rangle &= \tfrac{1}{\sqrt{2}}\langle \Lambda^0\pi^+ | \mathcal{H}\mathcal{I}_+ | \Sigma^{*0} \rangle = \tfrac{1}{\sqrt{2}}\langle \Lambda^0\pi^+ | \mathcal{I}_+\mathcal{H} | \Sigma^{*0} \rangle = \langle \Lambda^0\pi^0 | \mathcal{H} | \Sigma^{*0} \rangle \\ &= \tfrac{1}{\sqrt{2}}\langle \Lambda^0\pi^0 | \mathcal{H}\mathcal{I}_+ | \Sigma^{*-} \rangle = \tfrac{1}{\sqrt{2}}\langle \Lambda^0\pi^0 | \mathcal{I}_+\mathcal{H} | \Sigma^{*-} \rangle = \langle \Lambda^0\pi^- | \mathcal{H} | \Sigma^{*-} \rangle .\end{aligned}$$

It follows that all three decay rates are equal, so

$$\Gamma\left(\Sigma^{*+} \to \Lambda^0\pi^+\right) = \Gamma\left(\Sigma^{*0} \to \Lambda^0\pi^0\right) = \Gamma\left(\Sigma^{*-} \to \Lambda^0\pi^-\right).$$

5. Write the equation $\langle K^{*+}|[\mathcal{H},I_-]|K^+,\pi^+\rangle$ out explicitly, and use it to predict the relative decay rates $\Gamma\left(K^{*+}\to K^+\pi^0\right)$ and $\Gamma\left(K^{*+}\to K^0\pi^+\right)$. Then write out $\langle K^{*0}|[\mathcal{H},I_+]|K^0,\pi^-\rangle$, and use it to predict the relative decay rates of the K^{*0}.

Noting that $I_+|K^{*+}\rangle=0$, and because $[\mathcal{H},I_+]=0$, it follows that

$$0=\langle K^{*+}|[\mathcal{H},I_-]|K^+,\pi^+\rangle=\langle K^{*+}|\mathcal{H}I_-|K^+,\pi^+\rangle-\langle K^{*+}|I_-\mathcal{H}|K^+,\pi^+\rangle$$

$$=\langle K^{*+}|\mathcal{H}|K^0,\pi^+\rangle+\sqrt{2}\langle K^{*+}|\mathcal{H}|K^+,\pi^0\rangle-0,$$

$$\langle K^{*+}|\mathcal{H}|K^0,\pi^+\rangle=-\sqrt{2}\langle K^{*+}|\mathcal{H}|K^+,\pi^0\rangle.$$

Since decay rates are proportional to the magnitude squared of the matrix elements, it follows that

$$\Gamma\left(K^{*+}\to K^0\pi^+\right)=2\Gamma\left(K^{*+}\to K^+\pi^0\right).$$

In a similar manner, we note that $I_-|K^{*0}\rangle=0$ and $[\mathcal{H},I_-]=0$. It therefore follows that

$$0=\langle K^{*0}|[\mathcal{H},I_+]|K^0,\pi^-\rangle=\langle K^{*0}|\mathcal{H}I_+|K^0,\pi^-\rangle-\langle K^{*0}|I_+\mathcal{H}|K^0,\pi^-\rangle$$

$$=\langle K^{*0}|\mathcal{H}|K^+,\pi^-\rangle+\sqrt{2}\langle K^{*0}|\mathcal{H}|K^0,\pi^0\rangle-0,$$

$$\langle K^{*0}|\mathcal{H}|K^+,\pi^-\rangle=-\sqrt{2}\langle K^{*0}|\mathcal{H}|K^0,\pi^0\rangle,$$

and we therefore conclude that

$$\Gamma\left(K^{*0}\to K^+\pi^-\right)=2\Gamma\left(K^{*0}\to K^0\pi^0\right).$$