Solutions to Problems 9a

1. Based on the charge and strangeness of the octet of lightest baryons, deduce what quarks are inside them (don't worry about symmetrizing, just write the answer as $|p\rangle = |uud\rangle$). It is acceptable to repeat.

This is straightforward, and you did most of this in the reading quiz, so it is easy.

$$|p\rangle = |uud\rangle\,, \quad |n\rangle = |udd\rangle, \quad |\Sigma^+\rangle = |uus\rangle\,, \quad |\Sigma^0\rangle = |uds\rangle, \quad |\Sigma^-\rangle = |dds\rangle\,,$$
$$|\Lambda^0\rangle = |dds\rangle\,, \quad |\Xi^0\rangle = |uss\rangle\,, \quad |\Xi^-\rangle = |dss\rangle\,.$$

2. Although the 1, 8, and 10 dimensional representations of SU(3)$_F$ are all that are used for *light* quarks, there are some objects containing heavy quarks that are different. The Λ_c particle is a heavy spin $\frac{1}{2}$ baryon with quark content $|\Lambda_c\rangle = \frac{1}{\sqrt{2}}\left(|cud\rangle - |cdu\rangle\right)$. The charmed quark is unaffected by SU(3)$_F$, so $T_a|c\rangle = 0$ for all *a*.

(a) Suppose all the operators $T_{i\to j}$ are allowed to act repeatedly on $|\Lambda_c\rangle$. List all of the states that you can get. Make up names for them.

The operators $T_{i\to j}$ turn one type of quark into another, so we should probably rename them; for example, $T_{1\to 2}$ would be renamed $T_{u\to d}$. We find, for example, that

$$T_{u\to d}|\Lambda_c\rangle = \tfrac{1}{\sqrt{2}}\left(|cdd\rangle - |cdd\rangle\right) = 0\,, \quad T_{d\to u}|\Lambda_c\rangle = \tfrac{1}{\sqrt{2}}\left(|cuu\rangle - |cuu\rangle\right) = 0\,,$$
$$T_{u\to s}|\Lambda_c\rangle = \tfrac{1}{\sqrt{2}}\left(|csd\rangle - |cds\rangle\right) = |\Xi_c^0\rangle, \quad T_{d\to s}|\Lambda_c\rangle = \tfrac{1}{\sqrt{2}}\left(|cus\rangle - |csu\rangle\right) = |\Xi_c^+\rangle\,,$$
$$T_{s\to u}|\Lambda_c\rangle = T_{s\to d}|\Lambda_c\rangle = 0\,.$$

It is not hard to see that if we let the various generators act on these states, we don't get any additional states. We find

$$T_{u\to d}|\Xi_c^0\rangle = T_{u\to s}|\Xi_c^0\rangle = 0\,,$$
$$T_{d\to u}|\Xi_c^0\rangle = \tfrac{1}{\sqrt{2}}\left(|csu\rangle - |cus\rangle\right) = -|\Xi_c^+\rangle, \quad T_{d\to s}|\Xi_c^0\rangle = \tfrac{1}{\sqrt{2}}\left(|css\rangle - |css\rangle\right) = 0\,,$$
$$T_{s\to u}|\Xi_c^0\rangle = \tfrac{1}{\sqrt{2}}\left(|cud\rangle - |cdu\rangle\right) = |\Lambda_c\rangle, \quad T_{s\to d}|\Xi_c^0\rangle = \tfrac{1}{\sqrt{2}}\left(|cdd\rangle - |cdd\rangle\right) = 0,$$
$$T_{d\to u}|\Xi_c^+\rangle = T_{d\to s}|\Xi_c^+\rangle = 0\,,$$
$$T_{u\to d}|\Xi_c^+\rangle = \tfrac{1}{\sqrt{2}}\left(|cds\rangle - |csd\rangle\right) = -|\Xi_c^0\rangle, \quad T_{u\to s}|\Xi_c^+\rangle = \tfrac{1}{\sqrt{2}}\left(|css\rangle - |css\rangle\right) = 0\,,$$
$$T_{s\to u}|\Xi_c^+\rangle = \tfrac{1}{\sqrt{2}}\left(|cuu\rangle - |cuu\rangle\right) = 0\,, \quad T_{s\to d}|\Xi_c^+\rangle = \tfrac{1}{\sqrt{2}}\left(|cud\rangle - |cdu\rangle\right) = |\Lambda_c\rangle.$$

(b) Identify the T_3 and T_8 values for all the states you found in part (a) (including $|\Lambda_c\rangle$). Make a plot of T_3 vs. T_8 for these states.

The T_3 and T_8 values for each of these is just the sum of the corresponding eigenvalues for the two quarks (the charm quark doesn't contribute). We therefore have

$$T_3\left|\Lambda_c\right\rangle=\left(\tfrac{1}{2}-\tfrac{1}{2}\right)\left|\Lambda_c\right\rangle=0\,,\qquad T_8\left|\Lambda_c\right\rangle=\left(\tfrac{1}{2\sqrt{3}}+\tfrac{1}{2\sqrt{3}}\right)\left|\Lambda_c\right\rangle=\tfrac{1}{\sqrt{3}}\left|\Lambda_c\right\rangle,$$

$$T_3\left|\Xi_c^0\right\rangle=\left(0-\tfrac{1}{2}\right)\left|\Xi_c^0\right\rangle=-\tfrac{1}{2}\left|\Xi_c^0\right\rangle,\qquad T_8\left|\Xi_c^0\right\rangle=\left(\tfrac{1}{2\sqrt{3}}-\tfrac{1}{\sqrt{3}}\right)\left|\Xi_c^0\right\rangle=-\tfrac{1}{2\sqrt{3}}\left|\Xi_c^0\right\rangle,$$

$$T_3\left|\Xi^+\right\rangle=\left(0+\tfrac{1}{2}\right)\left|\Xi_c^+\right\rangle=+\tfrac{1}{2}\left|\Xi_c^0\right\rangle,\qquad T_8\left|\Xi^+\right\rangle=\left(\tfrac{1}{2\sqrt{3}}-\tfrac{1}{\sqrt{3}}\right)\left|\Xi_c^+\right\rangle=-\tfrac{1}{2\sqrt{3}}\left|\Xi_c^0\right\rangle.$$

The corresponding plot appears above right.

4. Draw at least thirteen tree-level Feynman diagrams for the process $q\bar{q}\to ggg$, where *q* is any quark, and *g* is a gluon. You don't have to do anything with the diagrams. For extra credit, find them all.

I have colored the three external gluon lines to help distinguish them from each other. For the seven diagrams with no an interior gluon line, I simply left that line black. There are six diagrams with three gluon/fermion vertices, six with two fermion/gluon vertex and a three gluon vertex, and with a single fermion/gluon vertex and a four gluon vertex, and three with a single fermion/gluon vertex and two three gluon vertices.