Midterm Exam Name: ____________________

Particle Physics, Fall 2012 October 16, 2012

Part I: Short Answer [50 points]

For each of the following, give a short answer (2-3 sentences, or a formula). [5 points each]

1. Explain qualitatively (a) how we accelerate particles to high energy, and (b) how we get them to go in a circle. Equations are nice but not necessary.

2. Suppose I were practicing archery, and I was trying to hit spherical water balloons of radius *R* with small arrows. What would be the cross-section for these targets?

3. Which of the following equations are manifestly Lorentz covariant?
   (a) $p_\mu \varepsilon^\mu p^\nu = p_\alpha p^\alpha \varepsilon^\nu$
   (b) $\tilde{F}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta} F_{\alpha\beta}$
   (c) $p^\mu q_\mu = p_\mu q^\mu$
   (d) $p^\mu = m$
   (e) $\varepsilon_{\mu\nu\alpha\beta} = -\varepsilon^{\mu\nu\alpha\beta}$

4. Explain physically or using equations what it means if your theory respects parity

5. Explain briefly, according to the Dirac theory, why it is that positive energy electrons do not normally "decay" and become negative energy electrons.

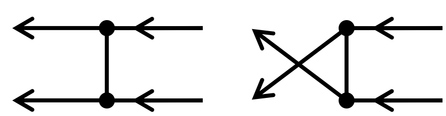
6. The unstable Z-boson is listed as having a mass of 91.19 GeV. This implies that for a Z at rest, $H\left|Z,\mathbf{p}=0\right\rangle = E\left|Z,\mathbf{p}=0\right\rangle$ with $E = 91.19\text{ GeV}$. Explain why we know for sure that, in fact, the Z-boson is not an eigenstate of the full Hamiltonian.

7. In weak interactions, you can get non-zero matrix elements of the form $\left\langle 0\left|\mathcal{H}\right|W^-,u,\bar{q}\right\rangle$, where W^- is the W-boson with charge –1, u is the up quark with charge +2/3, and $\bar{q}$ is an anti-quark of some sort. Tell me the charge of $\bar{q}$, and the charge of the corresponding quark q.

8. Suppose you are measuring the cross-section for some process, and you get very close to the mass of an intermediate particle (resonance). What happens, qualitatively, to the cross-section?

9. When you have a resonance, how can you tell from a graph of cross-section vs. energy what the decay rate or width Γ is for that intermediate particle?

10. In the Feynman diagrams at right, the arrow is a fermion, and the solid line is a boson. Would you add or subtract the contributions to the Feynman amplitude, and why?



Part II: Calculation [150 points]
Each problem has its corresponding point value marked. Solve the equations on separate paper.

11. [15] According to the particle data book, the K^+ meson has a mass of 493.7 MeV and a mean lifetime of $\tau = 1.238 \times 10^{-8}$ s
    (a) If a K^+ meson had an energy of E = 2350 MeV, how long would it last and how far would it go if it lasts one average lifetime?
    (b) What is the width Γ of a K^+, in eV?
    (c) The branching ratio for $K^+ \to \pi^+\pi^0$ is 20.7%. What is $\Gamma\left(K^+ \to \pi^+\pi^0\right)$, in eV?

12. [15] The differential cross-section for $e^+e^- \to \mu^+\mu^-$ at high energies is given by

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{16E^2}\left(1+\cos^2\theta\right)$$

    (a) Calculate the total cross-section
    (b) Convert to barns if E = 10.0 GeV. Use $\alpha = 1/137$.
    (c) If an e^+e^- collider is operating with each beam at E = 10.0 GeV at a luminosity of $L = 5.67\ \mu\text{b}^{-1}\text{s}^{-1}$, how many $\mu^+\mu^-$ pairs will it make in one day?

13. [10] A collision process takes the form $A(p_1)B(p_2) \to C(p_3)D(p_4)$, where the masses of the four particles are m_1, m_2, m_3, and m_4 respectively. Show that $p_2 \cdot p_4$ can be written in terms of $p_1 \cdot p_3$.

14. [20] Consider the process $\psi(p_1)\psi^*(p_2) \to \phi(k_1)\phi(k_2)$ in the center of mass frame. Let the mass of the ψ's be m and the mass of the ϕ's be M.
    (a) Assume the initial particles have energy E in the center of mass frame. Tell me the energy of the final particles, and the magnitude of the momentum of the initial and final particles. You must give arguments for your answers, not just the answers.
    (b) Write out explicitly all four components of all four momenta. You may assume the initial particles are coming in along the $\pm x^3$ axes. The final particles will go in an arbitrary direction.
    (c) Calculate all six dot-products of initial and final momenta explicitly, *i.e.*, tell me

$$p_1 \cdot p_2\,,\quad k_1 \cdot k_2\,,\quad p_1 \cdot k_1\,,\quad p_1 \cdot k_2\,,\quad p_2 \cdot k_1\,,\quad p_2 \cdot k_2\,.$$

15. [15] A high energy anti-neutrino $(m_\nu = 0)$ with energy E collides with an electron at rest ($m_e = 0.511$ MeV).
    (a) Find a simple formula for the center of mass energy squared s in terms of E and m_e.
    (b) How big must E be to produce a W boson ($m_W = 80.40$ GeV) via the process $\bar{\nu}_e e^- \to W^-$?

16. [25] Consider a renormalizable theory with two charged spin 0 particles, ψ_1 with charge +1, and ψ_3 with charge +3. They are not equivalent to their anti-particles ψ_1^* and ψ_3^*.

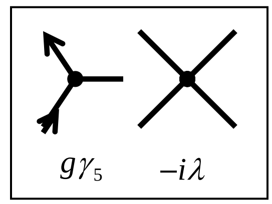
(a) Write down all possible renormalizable matrix elements of the form $\langle 0|\mathcal{H}|X\rangle$, where X has more than two particles, and figure out which ones must be real.

(b) Make up a diagrammatic notation for the particles ψ_1 and ψ_3. Draw all possible vertices for this theory, and give me the corresponding factor that should be included for this theory.

(c) Consider the scattering $\psi_3\psi_1^* \to \psi_1\psi_1$. Draw the relevant Feynman diagram, and give me the relevant Feynman amplitude. Then find the differential and total cross-section, treating all particles as massless, if the energy of each of the initial particles is E.

17. [25] Working in the full $\bar{\psi}\psi\phi$ theory with pseudoscalar couplings, sketch all seven tree-level diagrams for the process

$$\psi(p_1)\bar{\psi}(p_2) \to \phi(k_1)\phi(k_2)\phi(k_3)$$



Then, carefully write the Feynman invariant amplitude for two of them: one which does involve the λ coupling, and one which does not involve it. You don't have to do the other diagrams, nor do you have to do anything with the resulting amplitudes. The Feynman rules for the only two allowed vertices are given above.

18. [25] It is possible (though not likely) that one of the decays of the top quark will be

$$t(p_1, s_1) \to b(p_2, s_2)h^+(p_3),$$

where t and b are the top and bottom quark (both fermions) and h^+ is a charged scalar. Assume the amplitude for this process takes the form

$$i\mathcal{M} = \bar{u}_2(-i\alpha + \beta\gamma_5)u_1 ,$$

where α and β are real constants. Calculate the decay rate $\Gamma(t \to bh^+)$. Assume the top quark has mass m_t, the bottom quark has mass 0, and the scalar field has mass m_h.