Name ____________________
Solutions to Final Exam
May 9, 2012

This test consists of five parts. Please note that in parts II through V, you can skip one question of those offered. Some possibly useful formulas appear below.

### Part I: Multiple Choice (20 points)

For each question, choose the best answer (2 points each).

1. Two waves will interfere the most destructively, cancelling each other out, when their phases differ by
   - A) 0
   - B) $\pi$
   - C) $\frac{1}{2} \pi$
   - D) $\frac{3}{2} \pi$
   - E) $2 \pi$

2. When you look into the mirror, you see yourself on the other side because
   - A) The interference of photons causes a virtual you to be created behind the mirror
   - B) **Your brain interprets the combination of light rays coming back at you as if there were a person on the other side of the mirror**
   - C) Rays project out from your eyes, creating an image of yourself
   - D) The light rays have their polarization reversed, and your eyes are sensitive to this polarization and interpret it as someone on the other side
   - E) Your evil twin from an alternate universe is summoned and has to stand on the other side of the mirror
3. If parallel light shines on a diverging lens, the light that comes out will
   A) Remain parallel
   B) Pass through the far focus of the lens
   C) Pass through the far vertex of the lens
   D) Look as if it were coming from the near focus
   E) Look as if it were coming from the near vertex

4. Which of the following is one of Kirchoff’s Laws?
   A) The total current added up for all components in a loop has to equal zero
   B) The total voltage across all the components in a loop has to equal zero
   C) The voltages going into a vertex have to equal the voltages coming out
   D) The power consumed by a resistor has to match the power produced elsewhere in
      the circuit
   E) The voltage across an inductor is proportional to the current

5. If 8 C of charge flows through a wire in 2 s, the current must be
   A) 1/16 A  B) 1/4 A  C) 1/2 A  D) 2 A  E) 4 A

6. What is the correct way to arrange the two lenses in a simple telescope?
   A) Make them so they are as close together as possible
   B) Make them as far apart as possible
   C) Make sure the focal point of each lens is at the position of the other lens
   D) Make sure they share a common focal point
   E) Make sure the center of curvature of the two lenses correspond

7. Inductors resist changes in
   A) Voltage    B) Charge    C) Power    D) Resistance    E) Current

8. Why do simple imaging systems, like a single lens, have spherical aberration?
   A) Different colors refract by different amounts, and therefore don’t focus perfectly
   B) True lenses are never perfectly spherical, and therefore they don’t focus perfectly
   C) The angles in lenses are not always small, and hence focus only
      approximately
   D) No glass is perfectly transparent, and therefore some light is absorbed and blurred
   E) Glass is a mixture that is rarely uniform, so different parts of the lens bend light
      differently

9. Electrical power on a commercial scale is actually generated by
   A) Rotating loops of wire in magnetic fields
   B) Rotating loops of wire in electric fields
   C) Using powerful electric fields to push charges into the wire
   D) Using chemical reactions that move charges around
   E) Rubbing together disparate materials which separates the charges
10. Optical fibers can carry light signals for many kilometers without losing much power. How do we avoid having any light escape?
   A) The light beam is sent in so straight it never hits the sides
   B) A reflective coating causes the light to repeatedly reflect back into the fiber
   C) The high index glass in the center is surrounded by low index glass, so it totally internally reflects when it hits the sides
   D) A series of lenses are included in the fibers, constantly focusing the light back towards the center
   E) The fibers are simply so wide that very little light ever makes it to the sides of the fiber.

11. Diffraction effects when light goes through an aperture are greatest when
   A) The wavelength is long and the aperture is small
   B) The wavelength is short and the aperture is large
   C) The wavelength is long and the aperture is large
   D) The wavelength is short and the aperture is small
   E) On weekends and holidays

12. How does a diffraction grating divide light into its separate wavelengths?
   A) It refracts light by different amounts based on wavelength
   B) It absorbs certain colors in certain directions, and other colors in other directions
   C) It has many narrowly spaced lines which cause interference at different angles for different wavelengths
   D) It has atoms that absorb the light and reemit it, but at different angles for different wavelengths
   E) It measures the wavelength, thereby changing the momentum by a different amount for each color

13. When light falls on oil on top of water, you often get interesting color effects because
   A) Oil absorbs some colors and transmits others
   B) Oil’s index of refraction is highly dependent on wavelength, so different colors reflect at different angles
   C) Oil can convert light of one wavelength to another, depending on its thickness
   D) The air/oil and oil/water interface each produce reflections, and these will interfere, constructively for some colors and destructively for others
   E) Light converts oil into other substances that color the light

14. Which of the following are known to have both particle-like and wavelike properties?
   A) Light (only)
   B) Electrons (only)
   C) Atoms (only)
   D) Light and electrons, but not atoms
   E) Light, electrons, and atoms
15. An electromagnetic wave moving to the right \( \rightarrow \) can have which of the three polarizations sketched?  
A) 1 only  
B) 2 only  
C) 3 only  
D) 1 and 2  
E) 1 and 3

16. In order to avoid the ultraviolet catastrophe, Planck assumed which of the following?  
A) There is a maximum frequency that can be achieved at any given temperature  
B) Light energy comes in chunks that are proportional to the frequency  
C) Light energy comes in chunks that are proportional to the wavelength  
D) The system was not technically in thermal equilibrium; particularly the highest frequencies  
E) Uncertainty in the measurements resulted in the apparent discrepancy between theory and experiment

17. How can a mirror have a negative focal length?  
A) Whenever the mirror is concave  
B) Whenever the mirror is convex  
C) By shining the light on the back side of any mirror  
D) By reflecting from a low index of refraction to a high index of refraction  
E) By reflecting from a high index of refraction to a low index of refraction

18. The force on a moving charge in a magnetic field is  
A) Parallel to both the magnetic field and the velocity  
B) Parallel to the magnetic field and perpendicular to the velocity  
C) Perpendicular to the magnetic field and parallel to the velocity  
D) Perpendicular to both the magnetic field and the velocity  
E) None of the above

19. According to Lenz’s Law, when there is a change in magnetic flux, how does a current loop react?  
A) It creates a current that tries to keep the magnetic flux constant  
B) It creates a current that tries to keep the magnetic field constant  
C) It creates a current that tries to make the magnetic flux zero  
D) It creates a current that tries to make the magnetic field zero  
E) It creates a current that tries to make the electromotive force zero

20. If we sketch out the electric field lines, where, if anywhere, can they start and end?  
A) They can start from positive or negative charges, and can end only at infinity  
B) They can end on positive or negative charges, but must come from infinity  
C) They start from positive charges and end on negative charges  
D) They start from negative charges and end on positive charges  
E) Electric fields always form closed loops, or start and end at infinity
21. What sorts of electric fields can exist on the surface of a conductor?
   A) They can only be parallel to the surface of the conductor
   B) They can only be perpendicular to the surface of the conductor  
   C) There is no restriction; they can point any direction on the surface of a conductor
   D) They must vanish on the surface of the conductor
   E) They must form loops on the surface of the conductor

22. Which two things are proportional in a capacitor?
   A) Voltage and current
   B) Voltage and resistance
   C) Current and resistance
   D) Charge and resistance
   E) Voltage and charge

23. Which two components make an oscillator, which allows energy to flow back and forth between the two components?
   A) Inductor and Diode
   B) Inductor and Resistor
   C) Inductor and Capacitor
   D) Capacitor and Resistor
   E) Capacitor and Diode

24. How does the force of light falling perpendicular on an absorbing rectangle differ from the force of light falling on a reflective rectangle?
   A) The force vanishes for a reflective rectangle, since no light is absorbed
   B) The force vanishes for an absorbing rectangle
   C) The forces are equal
   D) The force for an absorbing rectangle is twice as large as for a reflecting rectangle
   E) The force for a reflecting rectangle is twice as large as for an absorbing rectangle

25. According to Gauss’s Law, we can calculate the total amount of charge inside a region if we know
   A) The total electric flux out of that region
   B) The total magnetic flux out of that region
   C) The total current out of that region
   D) The total electric potential in that region
   E) The electric field at one point on the surface of that region
Part II: Short answer, old material: [20 points]
Choose two of the following three questions and give a short answer (2-3 sentences) (10 points each)

26. What is the relation between the direction of electric field and the electric force on a charge $q$?

The force is given by $F = qE$. This means that positive charges feel a force in the direction of the electric field, and negative charges feel a force in the opposite directions. Neutral objects feel no force.

27. Draw an RC circuit and explain qualitatively what will happen if you put charge on the capacitor.

An RC circuit consists of a capacitor connected to a resistor, as sketched at right. When the resistor is connected to the capacitor, the charge starts to dissipate through the capacitor, causing the charge and voltage to fall off exponentially.

28. Explain qualitatively how Maxwell’s version of Ampere’s Law differs from the original Ampere’s Law.

Ampere’s Law says the integral of the magnetic field around a closed loop is proportional to the current. Maxwell’s version says there is an additional contribution coming from the change in the electric flux passing through the loop.
Part III: Short answer, new material: [20 points]

Choose three of the following four questions and give a short answer (2-3 sentences, or comparable) (10 points each)

29. When a light wave moves from air to water, which of the following increase, decrease, or stay the same: Index of Refraction, Frequency, Period, Wavelength, Wave Number, Speed.

Water has a higher index of refraction. Frequency (and therefore period) stays the same. The speed goes down, and since \( f \lambda = v \), the wavelength decreases. Wave number is inversely proportional to frequency, so \( k \) increases.

30. Give a qualitative description of how a Michelson Interferometer works.

A Michelson Interferometer divides a single light beam into two, sends them along two long paths, then recombines them. Even a slight shift in the length of one of the two paths will cause a phase shift, changing the interference and making even very small motions easily detectable.

31. What does polarization of light mean? Give at least two ways that you can produce partly or completely polarized light.

The polarization of light means the direction the electric field points. Normally light is mostly or partly randomized. You can polarize light by certain production mechanisms (such as from an antenna), by reflecting off of a medium at Brewster’s angle, by using a polarizer that only lets one polarization of light, or by using a birefringent crystal that separates the two polarizations.

32 Explain qualitatively the photoelectric effect, and how it demonstrates evidence for photons (quanta of light).

In the photoelectric effect, photons of light liberate electrons from a metal. It is observed that this takes effect immediately, even if the intensity is weak, but it only works if the frequency is sufficiently high. This can be easily explained if you assume they come in packets of energy \( E = hf \), and it takes some minimum energy \( \phi \) which is required to liberate them.
Part IV: Calculation, old material: [60 points]

Choose two of the following three questions and perform the indicated calculations (20 points each)

33. Two charges are arranged as sketched, with +6.00 \( \mu \)C at \( x = -3 \) cm and -4.00 \( \mu \)C at \( x = +3 \) cm.

(a) What is electric field (direction and magnitude) at the origin?

Electric field points away from positive charges and towards negative charges. This implies to the right from each of the charges. Hence we simply add the magnitudes of the electric field and don’t worry about the signs.

\[
E = \frac{k_1 |q_1|}{r_1^2} + \frac{k_2 |q_2|}{r_2^2} = \frac{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \left(6 \times 10^{-6} \text{ C} + 4 \times 10^{-6} \text{ C}\right)}{\left(3.00 \times 10^{-2} \text{ m}\right)^2} = 9.99 \times 10^7 \text{ N/C}.
\]

The direction is to the right.

(b) What is the electric potential at the origin?

Electric potential is given by \( k_2 q/r \), added for each of the charges. Since it is not a vector quantity, we just add up the contribution from each charge.

\[
V = \frac{k_1 q_1}{r_1} + \frac{k_2 q_2}{r_2} = \frac{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \left(6 \times 10^{-6} \text{ C} - 4 \times 10^{-6} \text{ C}\right)}{3.00 \times 10^{-2} \text{ m}} = 5.99 \times 10^7 \text{ N} \cdot \text{m/C}
\]

\( = 5.99 \times 10^5 \text{ V}. \)

(c) A test charge of magnitude 1.00 \( \mu \)C is placed at the origin. What is the electric force on this charge? How much total potential energy does it have?

We simply use the formulas \( \mathbf{F} = q \mathbf{E} \) and \( U = qV \) to obtain

\[
\mathbf{F} = q \mathbf{E} = \left(1.00 \times 10^{-6} \text{ C}\right) \left(9.99 \times 10^7 \text{ N/C}\right) = 99.9 \text{ N},
\]

\[
U = qV = \left(1.00 \times 10^{-6} \text{ C}\right) \left(5.99 \times 10^5 \text{ N} \cdot \text{m/C}\right) = 0.599 \text{ J}.
\]
34. Three resistors are connected as sketched at right to external leads.
(a) Find the effective resistance of the combination of the three resistors.

The 200 $\Omega$ and 400 $\Omega$ resistors are connected at one end, and hence in series, with an effective resistance of
\[ R_s = 200 \Omega + 400 \Omega = 600 \Omega. \]
If you now replace these two with an equivalent resistor, it is clear they are in parallel with the remaining one, so we have
\[ \frac{1}{R} = \frac{1}{300 \Omega} + \frac{1}{600 \Omega} = \frac{2+1}{600 \Omega} = \frac{1}{200 \Omega}. \]
Hence the total equivalent resistance is 200 $\Omega$.

(b) A 12 V battery is now connected across these resistors. Find the total current flowing from the battery.

Using the total resistance, we have $\Delta V = IR$, or solving for the current,
\[ I = \frac{\Delta V}{R} = \frac{12 \text{ V}}{200 \Omega} = 0.060 \text{ A}. \]

(c) Find the current through each of the three resistors.

Backing up slightly to the stage when the two resistors on the right were combined to yield $R_r = 600 \Omega$, we see that the circuit is essentially 12 V across each of two resistors, and we simply use $\Delta V = IR$ on them separately to yield
\[ I_{300} = \frac{\Delta V}{R} = \frac{12 \text{ V}}{300 \Omega} = 0.040 \text{ A} , \quad \text{and} \quad I_r = \frac{\Delta V}{R} = \frac{12 \text{ V}}{600 \Omega} = 0.020 \text{ A}. \]
Not surprisingly, the two add to 0.060 A, as they must. The current on the right feeds through both of the resistor on the right, so we have $I_{200} = I_{400} = I_r = 0.020 \text{ A}$.

(d) Find the power consumed in each of the three resistors, and the total power consumed.

The power consumed in each resistor is given by $P = I \Delta V = I^2 R$. We simply apply this separately to each resistor
\[ P_{300} = (300 \Omega)(0.040 \text{ A})^2 = 0.48 \text{ W} , \quad P_{200} = (200 \Omega)(0.020 \text{ A})^2 = 0.08 \text{ W} , \quad P_{400} = (400 \Omega)(0.020 \text{ A})^2 = 0.16 \text{ W}. \]
The total power can be computed by summing these to give $P_{\text{tot}} = 0.72 \text{ W}$. 
35. A loop of wire consists of 80 turns of wire in the shape of a rectangle. It is in a region with constant magnetic field \( B = 0.250 \, \text{T} \) out of the paper. The wire has a total length of 80.0 m, so that the rectangle is 1 m in total perimeter at all times. However, its length and width are constantly changing. The width in meters at time \( t \) (in seconds) between 0 and 1 seconds, is given by \( w = \frac{1}{2} t \), but the height shifts to compensate, so it starts as a tall, skinny rectangle and ends as a wide, short rectangle.

(a) Find a formula for the vertical length \( l \) as a function of time, and the area as a function of time.

The perimeter is given by \( P = 2l + 2w \), so since \( P = 1 \), we have \( 1 = 2l + 2w \) or \( \frac{1}{2} = l + w \). Solving for \( l \), we have

\[
l = \frac{1}{2} - w = \frac{1}{2} - \frac{1}{2} t.
\]

The area is \( A = lw \), so we have

\[
A = lw = \left( \frac{1}{2} - \frac{1}{2} t \right) \frac{1}{2} t = \frac{1}{4} t - \frac{1}{8} t^2.
\]

(b) Find a formula for the magnetic flux through a single loop at all times. Find the total flux through 80 loops.

We simply multiply by the magnetic field to get the flux, and then the number of turns in the loop to obtain

\[
\Phi_{B1} = AB = \left( \frac{1}{2} t - \frac{1}{8} t^2 \right) \left( \frac{1}{2} \right) = \frac{1}{4} t - \frac{1}{16} t^2, \quad \Phi_B = N\Phi_{B1} = 80 \left( \frac{1}{16} t - \frac{1}{16} t^2 \right) = 5t - 5t^2.
\]

(c) Find the EMF around the loop at all times. At what times between 0 and 1 seconds will this EMF cause a current to flow in the clockwise direction, and when will it be counter-clockwise?

The formula for EMF is

\[
\mathcal{E} = -\frac{d}{dt} \Phi_B = -\frac{d}{dt} \left( 5t - 5t^2 \right) = 10t - 5.
\]

This is the flux in the counter-clockwise direction. It is negative up to \( t = 0.5 \, \text{s} \), and then positive thereafter, so current would flow in the clockwise direction for the first half-second and counter-clockwise thereafter.
Part V: Calculation, new material: [60 points]

Choose three of the following four questions and perform the indicated calculations (20 points each).

36. A light beam enters a block of glass in the shape of an equilateral prism, at an angle of $30^\circ$ compared to normal.

(a) For red light, the index of refraction is $n = 1.51$. Find the angle $\theta_a$ it makes compared to normal as it enters the glass, the angle $\theta_b$ it makes as it prepares to leave the glass, and the angle $\theta_c$ after it leaves the glass.

The light must obey Snell’s Law, which tells us that

$$1 \cdot \sin 30^\circ = 1.51 \sin \theta_a,$$

$$\sin \theta_a = \frac{\sin 30^\circ}{1.51} = 0.3311,$$

$$\theta_a = \sin^{-1} 0.3311 = 19.337^\circ.$$ 

The triangle formed by the apex angle and the light ray has interior angles of $60^\circ$, $90^\circ - \theta_a$, and $90^\circ - \theta_b$, and since the interior angles of a triangle add to $180^\circ$, we have

$$\theta_b = 60^\circ - \theta_a = 60^\circ - 19.337^\circ = 40.663^\circ.$$ 

We use Snell’s law once more, and find

$$1 \cdot \sin \theta_c = 1.51 \sin 40.663^\circ = 0.9839,$$

$$\theta_c = \sin^{-1} 0.9839 = 79.7^\circ.$$ 

(b) For blue light, the index of refraction is $n = 1.53$. Repeat the three calculations you did in part (a). Hint – this part is a trick question. What actually happens to the light as it attempts to leave?

We simply repeat all the computations from before, changing the index of refraction as appropriate.

$$1 \cdot \sin 30^\circ = 1.53 \sin \theta_a,$$

$$\sin \theta_a = \frac{\sin 30^\circ}{1.53} = 0.3268,$$

$$\theta_a = \sin^{-1} 0.3268 = 19.075^\circ.$$

$$\theta_b = 60^\circ - \theta_a = 60^\circ - 19.075^\circ = 40.926^\circ.$$

$$1 \cdot \sin \theta_c = 1.53 \sin 40.926^\circ = 1.0023.$$ 

However, it is impossible to have sines greater than one, which implies the light is totally internally reflected. None of it actually escapes.
37. Below is sketched an imaging system consisting of a converging plano convex lens. The two foci of the lens are marked. Also sketched in are three light rays: An upper one, initially moving directly away from the near focus, a middle one, initially parallel to the optic axis, and the lower one, initially headed for the vertex of the lens.

(a) Sketch the path of each of the light rays after they pass through the lens.

The light ray through the vertex continues in a straight line (blue), the ray parallel to the optic axis will go through the far focus (green), and the ray that seems to come from the near focus will come out parallel (red).

(b) Using the paths just drawn, estimate the position of the image, and sketch it in.

The rays are obviously coming out diverging. To find the image, we trace them back in a straight line (dashed lines) and see where they intersect. We have sketched in the image (purple arrow).

(c) Using your metric ruler, measure the approximate focal distance $f$.

Assuming the lens is made of diamond with index of refraction $n = 2.419$, estimate the radius of the convex side of the lens.

I estimate the distance between the center of the lens and the focus at 6.3 cm. Using the lens-maker’s equation, we note that the front surface has a radius of infinity (because it is flat) and the back surface has a radius of $-R$, because it is concave as viewed from the front. The lensmaker’s equation then gives

$$ \frac{1}{6.3 \text{ cm}} = \left( \frac{2.419}{1} - 1 \right) \left( \frac{1}{\infty} + \frac{1}{R} \right) = \frac{1.419}{R}, $$

$$ R = 1.149 \cdot 6.3 \text{ cm} = 8.94 \text{ cm}. $$
38. The supergiant star Betelgeuse largest angular size of any star other than the Sun, \(2.70 \times 10^{-7}\) radians.

(a) The Hubble Telescope primary mirror has a diameter of 2.4 m. Assuming it is trying to resolve Betelgeuse using a visible wavelength of 580 nm, can it succeed?

The smallest angle that a telescope can resolve is approximately \(\theta_{\text{min}} = \lambda/d\), which gives us

\[
\theta_{\text{Keck}} = \frac{580 \times 10^{-9}}{2.4} \text{ m} = 2.42 \times 10^{-7}.
\]

Since this is slightly smaller than the angular size of Betelgeuse, it is in principle capable of resolving it, but just barely.

(b) By combining the signals of various radio telescopes scattered around the world, radio telescopes can effectively act as if they had a diameter as large as the diameter of the earth, about 12,700 km. If they are using radio waves at a frequency of 45 GHz \((4.5 \times 10^{10} \text{ s}^{-1})\), can radio astronomers resolve Betelgeuse?

We perform the same computation, but first we need the wavelength, which we get from the formula \(f\lambda = c\), so we have

\[
\lambda = \frac{c}{f} = \frac{2.998 \times 10^8 \text{ m/s}}{4.5 \times 10^{10} \text{ /s}} = 0.00666 \text{ m}.
\]

We then have

\[
\theta_{\text{Radio}} = \frac{0.00666 \text{ m}}{12.7 \times 10^6 \text{ m}} = 5.25 \times 10^{-10}.
\]

This is easily sufficient to resolve Betelgeuse.

(c) What is the energy of a single photon of wavelength 580 nm? What is the energy of a single photon of frequency 45 GHz?

We compute this using \(E = hf\), but first we need the frequency for the visible light. We can obtain this from \(f\lambda = c\), so we have

\[
f = \frac{c}{\lambda} = \frac{2.998 \times 10^8 \text{ m/s}}{580 \times 10^{-9} \text{ m}} = 5.17 \times 10^{14} \text{ s}^{-1}.
\]

We are now ready to get the energies in each case. I will work in eV:

\[
E_{\text{vis}} = hf = (5.17 \times 10^{14} \text{ s}^{-1})(4.136 \times 10^{-15} \text{ eV \cdot s}) = 2.138 \text{ eV} = 3.42 \times 10^{-19} \text{ J},
\]

\[
E_{\text{rad}} = hf = (4.50 \times 10^{10} \text{ s}^{-1})(4.136 \times 10^{-15} \text{ eV \cdot s}) = 1.86 \times 10^{-4} \text{ eV} = 2.98 \times 10^{-23} \text{ J}.
\]
39. If an object is in a box of size $a$, the quantum waves that fit in it will have wavelength $\lambda = 2a/n$, where $n$ is a positive integer.

(a) For each value of $n$, find a formula for the corresponding momentum $p$.

According to the de Broglie relation $p\lambda = h$, so we have

$$p = \frac{h}{\lambda} = \frac{h}{2a/n} = \frac{hn}{2a}.$$ 

That was easy.

(b) If the object has mass $m$, find a formula for the velocity $v$ and the kinetic energy $E$.

The velocity is the momentum over the mass, or

$$v = \frac{p}{m} = \frac{hn}{2am}.$$ 

The kinetic energy is then

$$E = \frac{mv^2}{2} = \frac{m}{2} \left( \frac{hn}{2am} \right)^2 = \frac{h^2n^2}{8a^2m}.$$ 

(c) If you have a box of size $a = 0.200$ nm, and you place an electron of mass $m = 9.11 \times 10^{-31}$ kg in it, what will be the smallest possible energy this electron can have in eV? This energy is called the zero-point energy.

The smallest energy comes from the smallest $n$, which is $n = 1$. We simply substitute all the numbers into our formula to give

$$E = \frac{h^2}{8a^2m} = \frac{\left( 6.626 \times 10^{-34} \text{ J} \cdot \text{s} \right)^2}{8 \left( 2.00 \times 10^{-10} \text{ m} \right)^2 \left( 9.11 \times 10^{-31} \text{ kg} \right)} = \left( 1.506 \times 10^{-18} \text{ J} \right) \frac{\text{eV}}{1.602 \times 10^{-19} \text{ J}} = 9.40 \text{ eV}.$$