Physics 744 - Field Theory

Homework Set 5

All three of these problems deal with the $\psi^*\psi\phi$ theory, containing a complex field ψ and a real field ϕ, with Lagrangian density

$$\mathcal{L} = \partial_\mu\psi^*\partial_\mu\psi + \tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi - m^2\psi^*\psi - \tfrac{1}{2}M^2\phi^2 - \gamma\psi^*\psi\phi$$

Note that for all but part of problem 1, the interaction term is irrelevant.

1. For this problem, treat the fields completely classically.
   (a) Write out the equations of motion for ψ, ψ^*, and ϕ. Verify that two of them are merely complex conjugates of each other.
   (b) Verify that $\psi \to e^{-i\theta}\psi$ is a symmetry of the theory. Work out the corresponding conserved current J_μ.

2. We now want to quantize the theory in the interaction picture.
   (a) Write the conserved quantity $Q = \int J^0(\vec{x})\, d^3\vec{x}$ in terms of the annihilation operators $\alpha_{\vec{k}}$, $\beta_{\vec{k}}$, and $\gamma_{\vec{k}}$ and their corresponding creation operators. For consistency, let $\alpha_{\vec{k}}$ and $\beta_{\vec{k}}$ annihilate the particle ψ and its corresponding anti-particle ψ^*, and let $\gamma_{\vec{k}}$ annihilate ϕ.
   (b) Write Q in terms of normalized, non-relativistic creation and annihilation operators, $a_{\vec{k}}$, $b_{\vec{k}}$ and $c_{\vec{k}}$. What is the total charge Q for a system containing n ψ's, m ψ^*'s and p ϕ's?

3. Work out expressions for all six of the free propagators given below, and write the answer in a manifestly Lorentz invariant manner (so it has an $\int d^4\mathbf{k}$ and a $\lim_{\varepsilon\to 0}$, as in class). Most of them will be trivially zero.

$$\langle 0|\mathcal{T}\left[\phi(\mathbf{x})\phi(\mathbf{y})\right]|0\rangle, \quad \langle 0|\mathcal{T}\left[\psi(\mathbf{x})\phi(\mathbf{y})\right]|0\rangle, \quad \langle 0|\mathcal{T}\left[\psi^*(\mathbf{x})\phi(\mathbf{y})\right]|0\rangle,$$
$$\langle 0|\mathcal{T}\left[\psi(\mathbf{x})\psi(\mathbf{y})\right]|0\rangle, \quad \langle 0|\mathcal{T}\left[\psi^*(\mathbf{x})\psi^*(\mathbf{y})\right]|0\rangle, \quad \langle 0|\mathcal{T}\left[\psi^*(\mathbf{x})\psi(\mathbf{y})\right]|0\rangle.$$