**Quantum Mechanics 742 – First Test**

**The following new equations you should have memorized:**

Variational Method:

$$E_g \le \frac{\langle\psi|H|\psi\rangle}{\langle\psi|\psi\rangle}$$

$$E(\boldsymbol{\alpha}) = \frac{\langle\psi(\boldsymbol{\alpha})|H|\psi(\boldsymbol{\alpha})\rangle}{\langle\psi(\boldsymbol{\alpha})|\psi(\boldsymbol{\alpha})\rangle}$$

$$E_g \approx E(\boldsymbol{\alpha}_{\min})$$

$$|\psi_g\rangle \approx \frac{|\psi(\boldsymbol{\alpha}_{\min})\rangle}{\sqrt{\langle\psi(\boldsymbol{\alpha}_{\min})|\psi(\boldsymbol{\alpha}_{\min})\rangle}}$$

Trace:

$$\mathrm{Tr}(A) \equiv \sum_i \langle\phi_i|A|\phi_i\rangle$$

$$\mathrm{Tr}(AB) = \mathrm{Tr}(BA)$$

State Operator

$$\rho \equiv \sum_i f_i |\psi_i\rangle\langle\psi_i|$$

$$\mathrm{Tr}(\rho) = 1, \quad \rho^\dagger = \rho$$

Eigenvalues: $\rho_i \ge 0$

$$\langle A\rangle = \mathrm{Tr}(\rho A)$$

Perturbation Theory: $H = H_0 + W$

$$E_n = \varepsilon_n + \langle\phi_n|W|\phi_n\rangle + \sum_{m\ne n}\frac{|\langle\phi_m|W|\phi_n\rangle|^2}{\varepsilon_n - \varepsilon_m}$$

$$|\psi_n\rangle = |\phi_n\rangle + \sum_{m\ne n}|\phi_m\rangle\frac{\langle\phi_m|W|\phi_n\rangle}{\varepsilon_n - \varepsilon_m}$$

WKB approximation: $n = 0,1,2,\ldots$

$$\int_a^b \sqrt{2m[E - V(x)]}\,dx = \left(n + \tfrac{1}{2}\right)\pi\hbar$$

Spin-Orbit Coupling Trick:

$$\mathbf{L}\cdot\mathbf{S} = \tfrac{1}{2}\left[(\mathbf{L}+\mathbf{S})^2 - \mathbf{L}^2 - \mathbf{S}^2\right]$$

$$(\mathbf{L}+\mathbf{S})^2 = \mathbf{J}^2 \to \hbar^2(j^2 + j)$$

**Other things you should know:**

- Many previous things, especially from last semester's midterm
- The time evolution operator is linear and unitary; this allows you to prove things from it
- How to use the propagator to get the wave function at time $t$ given it at time $t_0$
- How to get the state operator as a matrix, or to write it in terms of basis vectors
- How to tell if a state operator is legal; how to tell if it is a pure or mixed state
- How to evolve the state operator and use it for evaluating expectation values
- In the Heisenberg formalism, state vectors don't change, but operators do
- How to estimate the energy of the ground state in the variational method
- How to find the classical turning points $a$ and $b$ in the WKB method, and use it to estimate the energy
- How to divide a Hamiltonian into $H_0$ and a perturbation $W$
- How to estimate energies and eigenstates in non-degenerate perturbation theory
- When degenerate perturbation theory is needed
- How to figure out the leading (zeroth order) eigenstates and eigenenergies when you have degenerate perturbation theory
- How and why to change basis from eigenstates of operators like $L_z$ and $S_z$ to eigenstates of $\mathbf{J}^2$ and $J_z$ when dealing with perturbations of the form $\mathbf{L}\cdot\mathbf{S}$
- How to calculate eigenvalues of things like $\mathbf{L}\cdot\mathbf{S}$

**The following equations you need not memorize, but you should know how to use them if given to you:**

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|---|---|
| Propagator: $$\Psi(\mathbf{r},t)=\int d^3\mathbf{r}_0 K(\mathbf{r},t;\mathbf{r}_0,t_0)\Psi(\mathbf{r}_0,t_0)$$ $$K(\mathbf{r},t;\mathbf{r}_0,t_0)=\sum_n \phi_n(\mathbf{r})e^{-iE_n(t-t_0)/\hbar}\phi_n^*(\mathbf{r}_0)$$ | Free Propagator (1D): $$K(x,t;x_0,t_0)=\sqrt{\frac{m}{2\pi i\hbar(t-t_0)}}\exp\left[\frac{im(x-x_0)^2}{2\hbar(t-t_0)}\right]$$ |
| Heisenberg Picture $$\frac{d}{dt}A(t)=\frac{i}{\hbar}\left[H(t),A(t)\right]$$ | State Operators: $$i\hbar\frac{d}{dt}\rho=[H,\rho]$$ |
| | Spin-Orbit Coupling $$W_{\text{SO}}=\frac{g}{4m^2c^2}\frac{1}{r}\frac{dV_c(r)}{dr}\mathbf{L}\cdot\mathbf{S}$$ |