Physics 741 – Graduate Quantum Mechanics 1

Solutions to Chapter 11

1. [5] An electron in an unknown spin state $\left|a\right\rangle$ is brought into proximity with a second electron in a known spin state $\left|b\right\rangle$. We wish to make the spin of the second electron match the first. A *quantum Xerox device* will copy it onto the second spin, so $U_{\text{Xerox}}\left|a,b\right\rangle=\left|a,a\right\rangle$. A *quantum teleporter* will swap the two spin states, as $U_{\text{Teleport}}\left|a,b\right\rangle=\left|b,a\right\rangle$.

(a) [3] By considering the three initial spin states $\left|a\right\rangle=\left|+\right\rangle$, $\left|a\right\rangle=\left|-\right\rangle$, and $\left|a\right\rangle=\frac{1}{\sqrt{2}}\left(\left|+\right\rangle+\left|-\right\rangle\right)$, show that the quantum Xerox device is impossible.

If the quantum Xerox device exists, it must change the state $\left|+,b\right\rangle$ into $\left|+,+\right\rangle$ and $\left|-,b\right\rangle$ into $\left|-,-\right\rangle$, in other words

$$U_{\text{Xerox}}\left|+,b\right\rangle=\left|+,+\right\rangle \quad \text{and} \quad U_{\text{Xerox}}\left|-,b\right\rangle=\left|-,-\right\rangle$$

However, U_{Xerox} is a linear operator, and it follows that

$$U_{\text{Xerox}}\left[\tfrac{1}{\sqrt{2}}\left(\left|+,b\right\rangle+\left|-,b\right\rangle\right)\right]=\tfrac{1}{\sqrt{2}}\left(\left|+,+\right\rangle+\left|-,-\right\rangle\right).$$

However, the quantum Xerox device is supposed to evolve this state into

$$U_{\text{Xerox}}\left[\tfrac{1}{\sqrt{2}}\left(\left|+,b\right\rangle+\left|-,b\right\rangle\right)\right]=\tfrac{1}{\sqrt{2}}\left(\left|+\right\rangle+\left|-\right\rangle\right)\otimes\tfrac{1}{\sqrt{2}}\left(\left|+\right\rangle+\left|-\right\rangle\right)=\tfrac{1}{2}\left(\left|++\right\rangle+\left|+-\right\rangle+\left|-+\right\rangle+\left|--\right\rangle\right)$$

Obviously, these equations are inconsistent, and hence this is impossible.

(b) [2] By considering the same three initial states, show that the same problem does not apparently occur for the quantum teleport device.

The quantum teleport device should evolve the states according to

$$U_{\text{Teleport}}\left|+,b\right\rangle=\left|b,+\right\rangle \quad \text{and} \quad U_{\text{Teleport}}\left|-,b\right\rangle=\left|b,-\right\rangle$$

and therefore, by linearity,

$$U_{\text{Teleport}}\left[\tfrac{1}{\sqrt{2}}\left(\left|+,b\right\rangle+\left|-,b\right\rangle\right)\right]=\tfrac{1}{\sqrt{2}}\left(\left|b,+\right\rangle+\left|b,-\right\rangle\right)$$

But this is exactly what we would want it to do, so there is, in fact, no problem in this case.