Physics 742 – Graduate Quantum Mechanics 2

Solutions to Chapter 15

1. [10] An isolated tritium (hydrogen) atom ^{3}H has its electron in the ground state when it suddenly radioactively decays to ^{3}He, (helium) but the nucleus stays in the same place (no recoil). What is the probability that the atom remains in the ground state? What is the probability that it goes into each of the *n* = 2 states $|2lm\rangle$?

The probability is just $P(I \to F) = |\langle nlm|100\rangle|^2$, but the eigenstate of the initial Hamiltonian are *not* the same as the eigenstate of the final Hamiltonian. The angular integrals will vanish unless $l = m = 0$, and in this case the angular integral yields one, so we have

$$\langle nlm|100\rangle = \delta_{l0}\delta_{m0}\int_0^\infty r^2 R'_{n0}(r) R_{10}(r)\,dr$$

The prime doesn't denote derivative, but rather that the radial wave function must be evaluated for Helium, which has the same wave function except $a_0 \to \frac{1}{2}a_0$. We'll let Maple finish it for us, using the online wave functions for hydrogen.

```
> for n to 6 do integrate(r^2*subs(a=a/2,radial(n,0))
  *radial(1,0),r=0..infinity)^2 end do;
```

Just for fun, I worked out the first six solutions, with the probabilities listed below. The other $|2lm\rangle$ states, of course, have probability zero.

$$P(1s \to 1s) = \tfrac{512}{729},\quad P(1s \to 3s) = \tfrac{124416}{9765625},\quad P(1s \to 5s) = \tfrac{1166400000}{678223072849},$$
$$P(1s \to 2s) = \tfrac{1}{4},\quad P(1s \to 4s) = \tfrac{2048}{531441},\quad P(1s \to 6s) = \tfrac{243}{262144}.$$

If you add up all these probabilities, it comes to about 97.2%. Probably most of the remaining 2.8% represents the probability that the electron becomes unbound.