Physics 742 – Graduate Quantum Mechanics 2

Solutions to Chapter 18

4. [10] The present day density of electrons is about 0.21/m^3; however, these electrons are not all free.

(a) [5] Calculate the cross-section for free photon-electron scattering in m^2. If all the electrons *were* free, what would be the rate at which they scatter, today? How many scatterings would have occurred to an average photon in the age of the universe, 13.7 Gyr?

The equation for cross-section is the last equation of section F:

$$\sigma = \frac{8\pi\alpha^2\hbar^2}{3m^2c^2} = \frac{8\pi}{3}\left[\frac{\left(6.582\times10^{-16}\ \text{eV}\cdot\text{s}\right)\left(2.998\times10^{8}\ \text{m/s}\right)}{137.036\left(511{,}000\ \text{eV}\right)}\right]^2 = 6.653\times10^{-29}\ \text{m}^2\ .$$

The rate at which collisions occur is

$$\Gamma = n\sigma\left|\Delta\mathbf{v}\right| = \left(0.21\ \text{m}^{-3}\right)\left(6.653\times10^{-29}\ \text{m}^2\right)\left(2.998\times10^{8}\ \text{m/s}\right) = 4.188\times10^{-21}\ \text{s}^{-1}$$
$$= 1.32\times10^{-13}\ \text{y}^{-1}\ .$$

Multiplying by the age of the universe, we find a typical photon has had 0.0018 collision in the age of the universe. Of course, the universe used to be somewhat denser, but this gives us an idea of the probability of a relatively recent collision. Though a significant fraction of electrons *are* free, nowadays, this is a relatively recent occurrence. Most photons have not undergone any modern collisions.

(b) [5] The last time the electrons *were* free was when the universe was 1092 times smaller in all three directions, and it was only 380,000 years old. The number of electrons was about the same then (though the number *density* was much higher, since the universe was smaller). Redo part (a) at this time.

The rate will be higher by a factor of the increased density, which works out to

$$\Gamma' = \Gamma\left(1092\right)^3 = \left(1092\right)^3 1.32\times10^{-13}\ \text{y}^{-1} = 1.72\times10^{-4}\ \text{y}^{-1}$$

Multiplying by the age of the universe at this time, we conclude that a typical photon would have undergone about 65 recent collisions at this time. Because of the large number of collisions, the photons are assumed to be completely thermalized.