What science can there be more noble, more excellent, more admirably high and demonstrative, than this of the mathematics

—Benjamin Franklin.

1. Find the derivative of $f(x)$ if
   
   (a) $f(x) = x^3 \sin(2x)$.  
   (b) $f(x) = \int_0^{x^2} \tan(t) \, dt$  

2. (a) Evaluate the the indefinite integral $\int \frac{x^3 + 1}{\sqrt{x}} \, dx$;  
   
   (b) Find the value of the definite integral $\int_0^{\frac{\pi}{2}} \sin(x)\sqrt{3 + \cos(x)} \, dx$.  
   
   (c) What is the average value of $f(x) = \sin(x)\sqrt{3 + \cos(x)}$ on the interval $[0, \frac{\pi}{2}]$?  

3. Consider the region bounded by the curves $y = x + 3$ and $(y - 1)^2 = 3x + 10$ shown in the graph on the right.  

   (a) Set up, but do not evaluate, an integral for the area of the given region.  
   
   (b) Set up, but do not evaluate, an integral for the volume of the solid of revolution obtained by rotating the given region about the x-axis.  

4. Let $g(x) = \int_0^x f(t) \, dt$ for $0 \leq x \leq 6$, where the graph of $f$ is shown on the right.  

   (a) Find the intervals on which $g(x)$ is decreasing.  
   
   (b) Find the intervals on which the graph of $g(x)$ is concave upward. Do not fret over precise approximations; any reasonable answers will be accepted.  
   
   (c) Sketch a reasonable graph of $g(x)$ on $[0,6]$.  

Put your solutions on this page if possible; use back of this page or attach an additional page only if necessary. Turn these in at the beginning of the period on Wednesday. Do not discuss these questions with anyone until all papers have been turned in.

5. Given the function \( f(x) = \frac{x^3}{x^2 + 3x - 4} \). Find the first derivative of the function \( f \), and without looking at the graph of \( f \) or \( f' \), find the intervals on which the derivative \( f' \) is positive. You may use Maple to do all the algebraic calculations for you, but justify your conclusions carefully using one of two acceptable methods we talked about in class.

6. (a) Use the Midpoint Rule with \( n = 4 \) to approximate the value of the definite integral \( \int_{-1}^{1} \frac{1}{x^2 + 1} \, dx \). Be sure to write a sum which indicates you know how to use the Midpoint Rule before using Maple or a calculator to find the value of the sum to the nearest thousandth.

(b) Use the definition of definite integral as given in the book to write the definite integral in part (a) as a limit. Do not evaluate the limit.