Math 111: Final Exam Review Problems: Part Deux

December 2, 2016

1. Decide if the following is true or false. If $f(x)$ is a function with domain $(-\infty, \infty)$ then
$$\lim_{x \to 1} f(x) = f(1)$$

2. Decide if the following is true or false. If
$$\lim_{x \to a} f(x)$$
exist, then f is continuous at a.

3. If $k \neq 0$ is a constant determine if $f(x) = kx^{14} - 15kx + 2k$ has a root in $[-1, 1]$.

4. Choose the best answer. If f and g are continuous on $(-\infty, \infty)$ which of the following do not need to be continuous on $(-\infty, \infty)$?
a. $f(x) + g(x)$
b. $f(x)/g(x)$
c. $f(g(x))$
d. All of the above are continuous if f and g are continuous.

5. Sketch a graph of a function $f(x)$ satisfying the following properties:
$$\lim_{x \to 3} f(x) = 4 \text{ and } f(3) = 2$$
$$\lim_{x \to 4} f(x) = \infty \text{ and } f(4) = 1$$
$$\lim_{x \to 5} f(x) \text{ does not exist and } f(5) = 3$$
$$\lim_{x \to 6} f(x) = 0 \text{ and } f(6) \text{ does not exist.}$$

6. Choose the best answer. If f and g are continuous on $(-\infty, \infty)$ which of the following do not need to be continuous on $(-\infty, \infty)$?
a. $f(x) + g(x)$
b. $f(x)/g(x)$
c. $f(g(x))$
d. All of the above are continuous if f and g are continuous.

7. Differentiate the following functions with respect to the indicated independent variable:
   - $g(w) = 3w^2 \cos(w)$
   - $g(x) = \sqrt{2x^3 + \sqrt{x^2 + 1}}$
   - $h(s) = \dfrac{5s^4 - \tan(s)}{\sqrt[3]{2s^2 + 7}}$

8. Find the equation of the tangent line to the graph $y = \cos(x) + \sin(x)$ at the point $(\pi/2, 1)$.

9. Find y' if the following implicit relationship holds: $x^2 + xy + y^2 = 7$.

10. Find y' if the following implicit relationship holds: $\sin(xy) = \sin(x) + \sin(y)$.

11. For what value(s) of θ in $[0, 2\pi]$, is the tangent line to $f(\theta) = 2\sin(\theta) + \sin^2(\theta)$ horizontal?

12. For $f(x) = x^2 + 1$, find the two points for which the tangent line to the graph passes through the point (0,3).

13. For $f(x) = \begin{cases} x^2 & \text{if } x \leq 2 \\ mx + b & \text{if } x > 2 \end{cases}$, find all the values of m and b which make f differentiable everywhere.

14. Decide if the following is true or false. If $f''(c) = 0$ then $(c, f(c))$ is an inflection point of the curve $y = f(x)$.

15. Decide if the following is true or false. If $f'(c) = 0$, then f has a local minimum or maximum at c.

16. Decide if the following is true or false. There exists a function f such that $f(x) > 0$, $f'(x) < 0$ and $f''(x) > 0$ for all x.

17. Decide if the following is true or false. If f is differentiable everywhere, then all local minima and maxima of f occur at numbers c such that $f'(c) = 0$.

18. If f has an absolute maximum value at c then $f'(c) = 0$.

19. If f is increasing on (a, b) then $f'(x) > 0$ for all x in (a, b).

20. Decide if the following is true or false. If a continuous function satisfies $f(1) = f(3)$, then there is a critical point of f between $x = 1$ and $x = 3$.

21. Decide if the following is true or false. The product rule says that if F is an antiderivative of f and G is an antiderivative of g, then the antiderivative of fg is $Fg + fG$.

22. A particle moves with acceleration $a(t) = \sin(t) + t^2$ meters per second squared. At $t = 0$ the particle initially has a velocity of 2 meters per second and a position of 3 meters. Find its position as a function of time.

23. A particle is moves at an acceleration $a(t) = kt$, where $k > 0$ is a constant. If the particle accelerates from 2 meters per second to 8 meters per second in 30 seconds find the value of k.

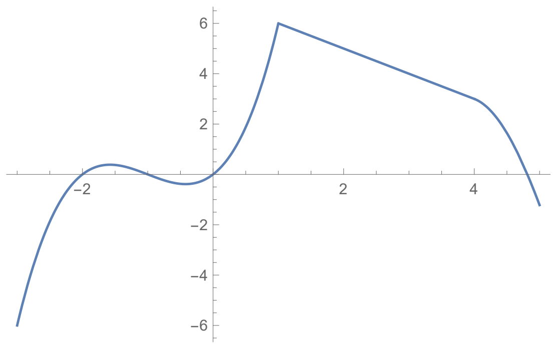
24. Consider a cube whose side length is increasing at a rate of 3 inches per second. At what rate is its surface area increasing when its side length is 6 inches.

25. Consider the function $f(x) = (1 + 2x)^{1/4}$ and let $x_0 = 0$.
    - Find a linear approximation to $f(x)$ near x_0.
    - Use your answer to estimate the solution of the equation $f(x) = 0$.
    - What is the exact solution of the equation $f(x) = 0$?

26. State Rolle's Theorem. Explain what it means and why it is true. Illustrate with a picture.

27. Find the antiderivative $F(x)$ of $f(x) = \sec^2(x) + \sec(x)\tan(x)$ that satisfies $F(0) = 0$.

28. Below is the graph of a function $f(x)$. Using the graph, sketch a graph of the antiderivative $F(x)$ of $f(x)$ that goes through the origin.



29. Evaluate the following integral:
$$\int_4^9 \frac{x^2 - \sqrt{x}}{x^{5/2}}\, dx.$$

30. What is the antiderivative of
$$f(x) = \frac{x-2}{(x^2-4x+8)^2}?$$

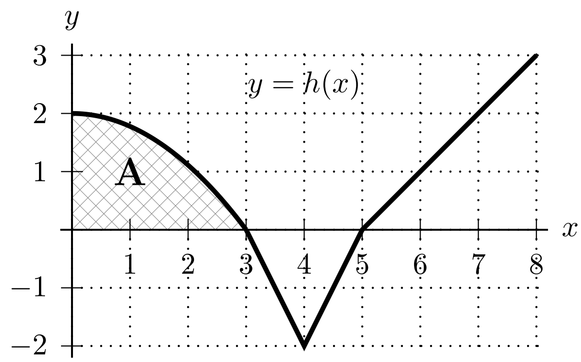
31. Suppose the acceleration of a rocket launched from earth is given by $a(t) = 360t$ meters per second squared t seconds after launch. The rocket is being sent to the International Space Station which is 240km above the surface of the Earth. If the rocket is launched from the ground and has zero initial velocity, how long does it take the rocket to reach the International Space Station?

32. Compute the following indefinite integral
$$\int \sqrt{1+\sqrt{x}}\, dx.$$

33. Evaluate the following integral:
$$\int_0^1 \frac{x^2-3x+2}{(x-2)(x+3)^3}\, dx.$$

The graph of a function $h(x)$ is shown below. The area of the shaded region A is 4, and $h(x)$ is piecewise linear for $3 \le x \le 6$.

- Find $\displaystyle\int_0^3 (h(x)+2)\, dx$.
- Find $\displaystyle\int_0^4 h(2x)\, dx$.
- Let $J(x) = \sin(\pi h(x))$. Find $J'(3.5)$.
- Let $H(x)$ be an antiderivative of $h(x)$ with $H(4) = 5$. Find an equation for the tangent line to the graph of $H(x)$ at $x = 4$.



34. Let $f(x)$ and $g(x)$ be increasing continuous functions defined on the interval $[0, 10]$ with $f(0) = g(0) = 0$. Also suppose f is always concave down and g is always concave up. For each of the following statements, determine whether it is always true, sometimes true, or never true.

   - $\displaystyle\int_0^{10} f(x)\,dx > \int_0^{1} 0g(x)\,dx$.
   - $f'(10) < g'(10)$.
   - $g'(0) > g'(2)$.
   - $\displaystyle\int_0^{10} |f(x)|\,dx = \int_0^{10} f(x)\,dx$.
   - $\displaystyle\int_0^{10} f'(x)\,dx > 0$.
   - If G is an antiderivative of g, then $G(10) > 0$.

35. The functions f and g have continuous second derivatives. The table below gives values of the functions and their derivatives at selected values of x.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	–6	3	2	8
2	2	–2	–3	0
3	8	7	6	2
6	4	5	3	–1

- Let $k(x) = f(g(x))$. Write an equation for the line tangent to the graph of k at $x = 3$.
- Let $h(x) = \dfrac{g(x)}{h(x)}$. Find $h'(1)$.
- Evaluate $\displaystyle\int_1^3 f''(2x)\,dx$.

36. Suppose $H(x) = \displaystyle\int_0^{x^3} x(1+x)^{\frac{4}{3}}\,dx$.

- Find all of the critical points of $H(x)$.
- Find all of the inflection points of $H(x)$.