Math 111
Fall 2016
Final Exam Practice
12/09/16

Name (Print): ____________________

The following rules apply:

- **If you use a "fundamental theorem" you must indicate this** and explain why the theorem may be applied.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Short answer questions:** Questions labeled as **"Short Answer"** can be answered by simply writing an equation or a sentence or appropriately drawing a figure. No calculations are necessary or expected for these problems.
- **Unless the question is specified as short answer, mysterious or unsupported answers might not receive full credit**. An incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.

Do not write in the table to the right.

Problem	Points	Score
1	15	
2	25	
3	15	
4	20	
5	20	
6	25	
7	20	
8	20	
9	20	
10	10	
11	25	
12	20	
13	20	
14	20	
15	10	
16	15	
Total:	300	

1. (15 points) (**Short Answer**) Determine if the following statement is correct (**C**) or incorrect (**I**). Just circle **C** or **I**. No need to show any work. In order for a statement to be correct it must be true in all cases.

   C **I** If f is continuous on the closed interval $a \leq x \leq b$, then f has a global maximum and global minimum on that interval.

   C **I** If $g''(5) = 0$ then $g(x)$ must have an inflection point at $x = 5$.

   C **I** If $f(x) = 10^x$ then $f'(a) \leq \dfrac{f(b) - f(a)}{b - a}$.

   C **I** If $f(x)$ is defined and differentiable on $[a, b]$ and $f(x)$ has an absolute maximum at $x = c$ in the interval (a, b) then $f'(c) = 0$.

   C **I** If $f'(x)$ has three roots in an interval $[a, b]$ then $f(x)$ has four roots in $[a, b]$.

2. (25 points) A table showing some values for the derivative of a function h with continuous first derivative is given below. Assume that h only increases or only decreases between consecutive values of x. For each statement, indicate whether the statement is true, false, or cannot be determined from the given information.

x	-4	-3	-2	-1	0	1	2	3	4
$h'(x)$	2	3	1	-3	-4	-2	0	2	1

   (a) (5 points) The function h has a local maximum on the interval $-2 < x < 1$.

   True False Not enough information

   (b) (5 points) The function h is negative on the interval $-1 < x < 1$.

   True False Not enough information

   (c) (5 points) The function h is concave up on the interval $0 < x < 4$.

   True False Not enough information

   (d) (5 points) The function h has an inflection point on the interval $-1 < x < 1$.

   True False Not enough information

   (e) (5 points) The function h is decreasing on the interval $-3 < x < -2$.

3. (15 points) Let $g(x)$ be a function for all values of x with

$$g(-1) = 5, \quad g(1) = 4, \quad g'(-1) = -2, \quad g'(1) = 3.$$

(a) (5 points) Find $m'(1)$ if $m(x) = \sqrt{1 + g(x)}$.

(b) (5 points) Find $p'(-1)$ if $p(x) = g(x)g(x^5)$.

(c) (5 points) Find $y'(1)$ if $y(x) = \dfrac{\sin^2(x)}{g(x) + 2}$.

4. (20 points) Find the indicated derivative for each function.

(a) (5 points) $f'(x)$ for $f(x) = \dfrac{1}{\sin(5x)}$.

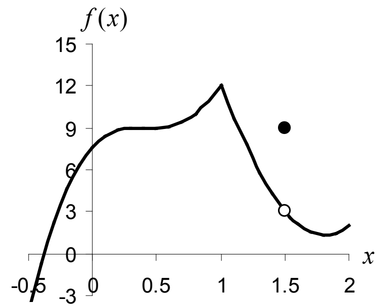
(b) (5 points) $g''(t)$ for $g(t) = at^3 + bt^2 + c^2$.

(c) (5 points) $\dfrac{d}{dz}\dfrac{8z+1}{7z-3}$.

(d) (5 points) $\dfrac{dy}{dr}$ for $y = 3\sqrt{5-r^2}$.

5. (20 points) Consider the family of functions defined by $f(t) = \dfrac{at}{t^2 + b}$ where $a > 0$, $b > 0$. Find all critical points of $f(t)$ then classify the critical points as local maxima, local minima, or neither. Show all your work clearly.

6. (25 points) Find the following limits, if they exist. Show all work for parts (d) and (e).



(a) (5 points) $\lim_{x \to 1} f(x)$.

(b) (5 points) $\lim_{x \to 1} f'(x)$.

(c) (5 points) $\lim_{x \to 1.5} f(x)$.

(d) (5 points) $\lim_{t \to 0^+} \dfrac{\sin(t)}{\sqrt{t}}$.

(e) (5 points) $\lim_{h \to 0} \dfrac{\sqrt{5+h} - \sqrt{5}}{h}$.

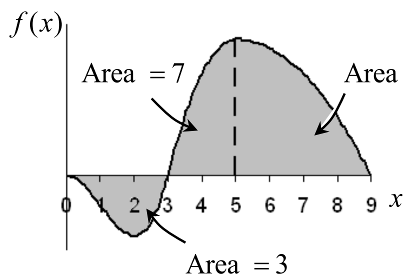
7. (20 points) Find the equation of the tangent line to the curve $y^3 - 2x^3 + 2a^3 = x^2y$ at the point (a, a).

8. (20 points) Suppose the height of a triangle decreases at a rate of 5 feet per minute while the area of the triangle increases at a rate of 8 square feet per minute. At what rate is the base of the triangle changing when the height is 12 feet and the area is 72 square feet.

9. (20 points) The National Aquarium in Baltimore has decided to make a new jellyfish tank. It will be cylindrical with a round base and top. The sides will be made of a special type of glass which costs \$80 per square meter, and the materials for the top and bottom of the tank cost \$45 per square meter. If the tank must hold 60 cubic meters of water, determine the radius that will minimize the total cost. Verify that your solution is actually a minimum.

10. (10 points) Find an antiderivative $G(x)$ with $G'(x) = g(x)$, $G(1) = 0$, and $g(x) = \dfrac{3+2x}{x}$.

11. (25 points) Use the graph of $f(x)$ shown below to answer the following. Assume $F'(x) = f(x)$.



(a) (5 points) $\int_0^9 2f(x)\,dx$.

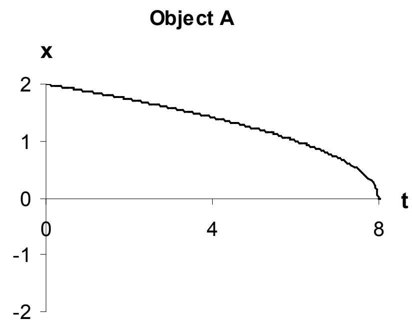
(b) (5 points) $\int_0^5 |f(x)|\,dx$.

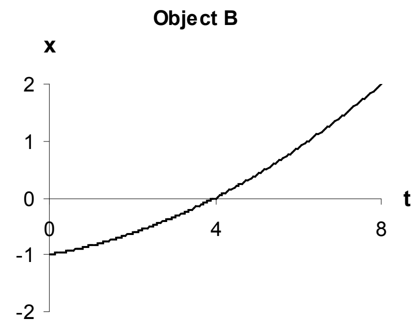
(c) (5 points) $\int_0^5 (f(x) - 4)\,dx$.

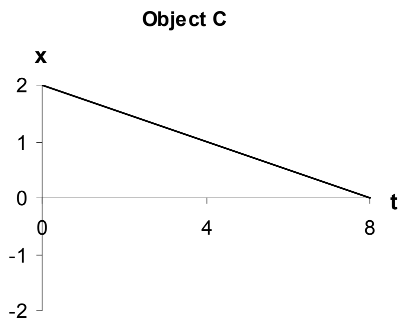
(d) (5 points) $F(9) - F(3)$.

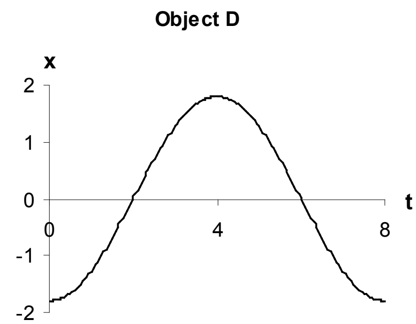
(e) (5 points) If a left-hand Riemann sum with $n = 4$ is used to estimate $\int_5^9 f(x)\,dx$, would it produce an underestimate or an overestimate.

12. (20 points) The following graphs show the **position** of four objects moving along the x-axis as a function of time, $0 \leq t \leq 8$. The questions below refer only to this time interval. Time is measured in seconds. If your answer to the question is none, write NONE.









(a) (5 points) Which object ends up farthest from its starting point?

(b) (5 points) Which object, if any, has a decreasing velocity throughout the first 8 seconds?

(c) (5 points) Which object, if any, has zero acceleration throughout the first 8 seconds?

(d) (5 points) Which object, if any, is at the origin at 4 seconds?

13. (20 points) Consider a family of functions $f(x)$ with positive parameters A, B and k. The derivative of this family is given by $f'(x) = \sqrt{1+kx^2}(Ax - B)$.

    (a) (10 points) Find the critical points of $f(x)$.

    (b) (10 points) Use the Second Derivative Test to classify the critical point(s) of $f(x)$ in part (a) as a local maximum or minimum.

14. (20 points) Find each of the following (continuous on next page)

(a) (5 points) $\displaystyle\int \left(\sec^2(\theta) - 4\right)\, d\theta.$

(b) (5 points) $\displaystyle\int \frac{4x^3+1}{x^3}\, dx.$

(c) (5 points) $\displaystyle\int \frac{t}{at^2+b}\, dt.$

(d) (5 points) $\displaystyle\int_0^1 \frac{y^4}{(7+y^5)^{1/3}}\, dy.$

15. (10 points) Oil is leaking out of a ruptured tank at a rate, $r(t)$, measured in hundreds of gallons per minute. Some values of $r(t)$ during the first 15 minutes are given in the table below

t	0	5	10	15
$r(t)$	3	11	24	43

(a) (5 points) Estimate the amount of oil that leaked out of the tank during the first 15 seconds.

(b) (5 points) Is your estimate an underestimate, overestimate, or you can't tell.

16. (15 points) Evaluate the following limit by recognizing it is the limit of a Riemann sum and using the fundamental theorem of calculus

$$\lim_{n\to\infty}\sum_{i=1}^{n}\sin\left(\frac{\pi}{2}+i\frac{\pi}{n}\right)\frac{\pi}{n}$$