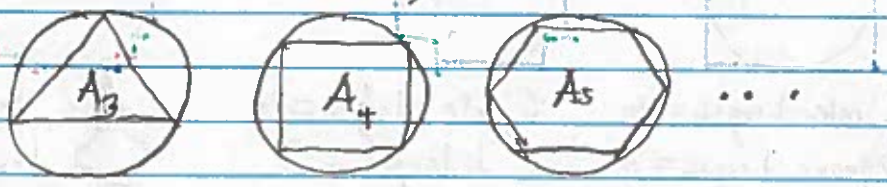
Lecture 0: What is Calculus?

What is π?

→ circle of radius 1



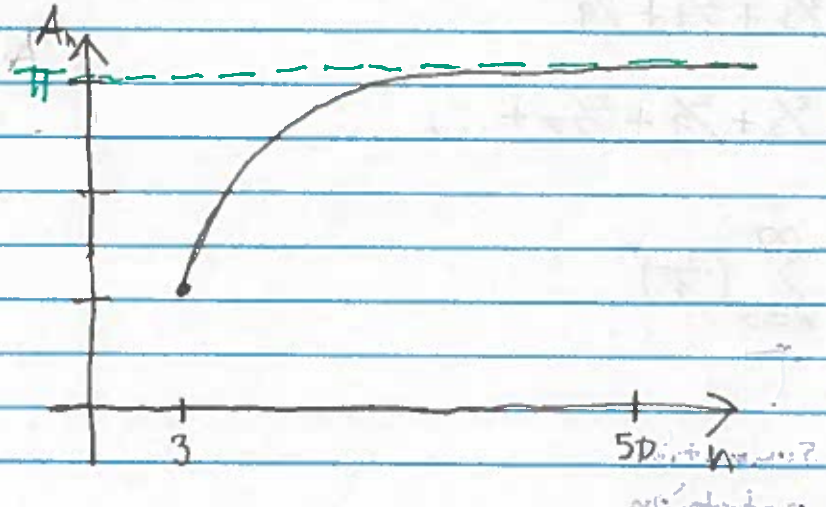
$$A_n = n \cdot \cos\left(\frac{(n-2)\pi}{2n}\right) \sin\left(\frac{(n-2)\cdot\pi}{2n}\right)$$

$A_3 = \frac{3\sqrt{3}}{4} \approx 1.299$, $A_5 \approx 2.378$, $A_7 = 2.736$

$A_4 = 2$, $A_6 \approx 2.598$, $A_8 = 2.83$

⋮

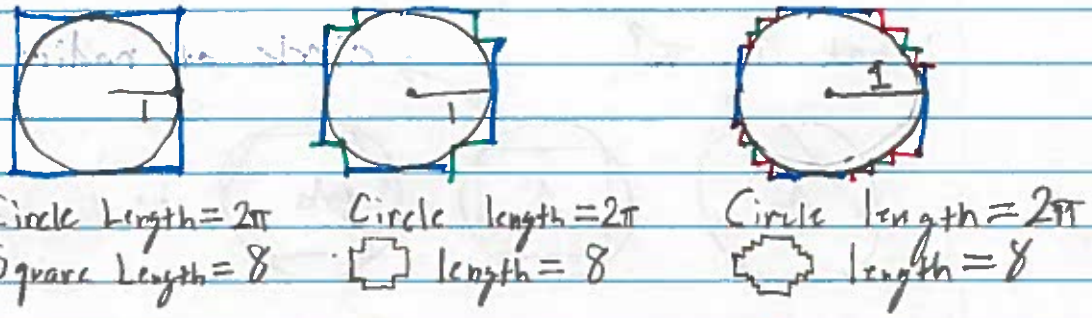
$A_{50} = 3.1333$
$A_{100} = 3.13953$
$N_{1000} = 3.14157$



As n gets large $A_n \approx \pi$.

$$\lim_{n\to\infty} A_n = \pi \approx 3.14159\ldots$$

What is π?

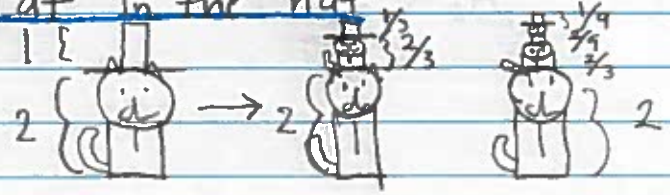


Circle Length $= 2\pi$ | Circle length $= 2\pi$ | Circle length $= 2\pi$
Square Length $= 8$ | length $= 8$ | length $= 8$

"Take limits" $\Rightarrow 8 = 2\pi \Rightarrow \pi = 4$!!

Something is clearly wrong. Calculus is the precise mathematical theory of limits.

Cat in the hat



$3 = 1 + 2$

$3 = 2 + \frac{1}{3} + \frac{2}{3}$

$3 = 2 + \frac{2}{3} + \frac{2}{9} + \frac{1}{9}$

$\vdots$

$3 = 2 + \frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \ldots$

$\vdots$

$$3 = 2\sum_{n=0}^{\infty}\left(\frac{1}{3}\right)^n$$

Summation notation