1 Problems for everybody

1. Let $\vec{u} \in \mathbb{R}^n$ and $\vec{v} \in \mathbb{R}^n$. Prove that $\vec{u}\vec{v}^T$ is a rank one matrix.

2. If $p(x) = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \ldots + x^{n-1}$, find the matrix representation of the operator defined by

$$L(p(x)) = \int_1^x p(x) \, dx.$$ 

3. Give an example of a matrix $A \in \mathbb{R}^{4 \times 4}$ such that $\text{Range}(A) = \text{Null}(A)$. Prove that there does not exist a matrix $A \in \mathbb{R}^{5 \times 5}$ such that $\text{Range}(A) = \text{Null}(A)$.

4. Problems 1.1, 1.3, 1.4.