1 Problems for everybody

1. Suppose $a, b \in \mathbb{R}$. Show that $\Lambda = \begin{bmatrix} a + bi & 0 \\ 0 & a - bi \end{bmatrix}$ is similar to $M = \begin{bmatrix} a & b \\ -b & a \end{bmatrix}$ by diagonalizing $M$.

How would you geometrically describe the linear transformation given by $M$?

2. Prove that the eigenvalues of a projector $P$ can have no other value than zero or one.

3. #24.1, #24.4.

2 MST Graduate student problems

1. The Fibonacci sequence $F_1, F_2, \ldots$ is defined by

$F_1 = 1$, $F_2 = 1$, and $F_n = F_{n-2} + F_{n-1}$ for $n \geq 3$.

Define $A \in \mathbb{R}^{2 \times 2}$ by its action: $A \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} y \\ x + y \end{bmatrix}$.

- Show that $A^n \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} F_n \\ F_{n+1} \end{bmatrix}$ for each positive integer $n$.
- Find the eigenvalues and eigenvectors of $A$.
- By using your solution to the previous bullet point show that

$F_n = \frac{1}{\sqrt{5}} \left[ \left( \frac{1 + \sqrt{5}}{2} \right)^n - \left( \frac{1 - \sqrt{5}}{2} \right)^n \right]$. 