Homework 9

Numerical Linear Algebra

November 3, 2017

1 Problems for everybody

1. Suppose $a, b \in \mathbb{R}$. Show that $\Lambda = \begin{bmatrix} a+bi & 0 \\ 0 & a-bi \end{bmatrix}$ is similar to $M = \begin{bmatrix} a & b \\ -b & a \end{bmatrix}$ by diagonalizing M. How would you geometrically describe the linear transformation given by M?
2. Prove that the eigenvalues of a projector P can have no other value than zero or one.
3. #24.1, #24.4.

2 MST Graduate student problems

1. The Fibonacci sequence $F_1, F_2, \ldots$ is defined by

$$F_1 = 1, \; F_2 = 1, \; \text{and } F_n = F_{n-2} + F_{n-1} \text{ for } n \geq 3.$$

Define $A \in \mathbb{R}^{2\times 2}$ by its action: $A \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} y \\ x+y \end{bmatrix}$.

- Show that $A^n \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} F_n \\ F_{n+1} \end{bmatrix}$ for each positive integer n.
- Find the eigenvalues and eigenvectors of A.
- By using your solution to the previous bullet point show that

$$F_n = \frac{1}{\sqrt{5}} \left[\left(\frac{1+\sqrt{5}}{2} \right)^n - \left(\frac{1-\sqrt{5}}{2} \right)^n \right].$$