Homework #7

#1.

Let $\vec{x},\vec{y}\in\mathbb{R}^n$, $A\in\mathbb{R}^{m\times n}$, $B\in\mathbb{R}^{n\times p}$. Find asymptotic formulas for the flops for the following calculations.

- $\vec{x}^T\vec{y}$
- $A\vec{x}$
- AB.

Solution:

1. $\vec{x}^T\vec{y}=\sum_{i=1}^{n} x_i y_i \sim 2n$ flops since there are $\sim n$ multiplications and $\sim n$ additions.
2. $A\vec{x}=\sum_{i=1}^{m}\sum_{j=1}^{n}(A_{ij}x_j)\vec{e}_i \sim 2mn$ flops since for each row the are $\sim 2n$ flops and there are m rows.
3. $(AB)_{ij}=\sum_{k=1}^{n} A_{ik}B_{kj}\sim 2n$ flops. Since AB has $m\cdot p$ entries it follows that $A\cdot B\sim 2mnp$ flops.

#4

The dominant contribution is:

$$U(j,k:n)=\underbrace{U(j,k:n)-L(j,k)\cdot \underbrace{U(k,k:n)}_{n-k\text{ multiplications}}}_{n-k\text{ subtractions}}$$

$$\Rightarrow \text{flops} \sim \sum_{k=1}^{n-1}\sum_{j=k+1}^{n} 2(n-k)$$

$$\sim \sum_{k=1}^{n-1} 2(n-k)\cdot j\Big|_{j=k+1}^{j=n}$$

$$\sim \sum_{k=1}^{n-1}(2n^2-4kn+2k^2)$$

$$\sim 2n^2k-2k^2n+\frac{2}{3}k^3\Big|_{k=1}^{k=n-1}$$

$$\sim \frac{2}{3}n^3.$$

#5.

The dominant contribution is:

$$R(j,j:n) = \underbrace{R(j,j:n) - \underbrace{R(k,j:n) \cdot R(k,j)/R(k,k)}_{n-j \text{ multiplications.}}}_{n-j \text{ subtractions.}}$$

$$\Rightarrow \text{flops} \sim \sum_{k=1}^{n} \sum_{j=k+1}^{n} 2(n-j)$$

$$= \sum_{k=1}^{n} \sum_{j=k+1}^{n} 2n - \sum_{k=1}^{n} \sum_{j=k+1}^{n} 2j$$

$$\sim \sum_{k=1}^{n} 2nj \Big|_{j=k+1}^{j=n} - \sum_{k=1}^{n} j^2 \Big|_{j=k+1}^{j=n}$$

$$\sim \sum_{k=1}^{n} (2n^2 - 2nk) - \sum_{k=1}^{n} (n^2 - k^2)$$

$$\sim \left(2n^2k - nk^2 - n^2k + \frac{k^3}{3}\right)\Big|_{k=1}^{k=n}$$

$$\sim 2n^3 - n^3 - n^3 + \frac{n^3}{3}$$

$$= \frac{n^3}{3}.$$