Quiz #2
MTH 351/651: Fall 2025
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In a certain chemical reaction, the concentration $A(t)$ of a substance satisfies

$$\frac{dA}{dt} = k_1 - k_2 A - k_3 A^2,$$

where $k_1, k_2, k_3 > 0$ are all constants. Letting $a = \frac{k_2}{k_1} A$ and $\tau = k_2 t$, show that this system is equivalent to the following dimensionless differential equation

$$\frac{da}{d\tau} = 1 - a - ka^2,$$

for some dimensionless constant k.

$$\frac{d}{dt} = \frac{d\tau}{dt}\frac{d}{d\tau} = k_2 \frac{d}{d\tau}$$

Therefore,

$$\frac{dA}{dt} = \frac{d}{dt}\left(\frac{k_1}{k_2} a\right) = \frac{k_1}{k_2}\frac{da}{dt} = \frac{k_1}{k_2} k_2 \frac{da}{d\tau}$$

$$\Rightarrow k_1 - k_2 \cdot \frac{k_1}{k_2} a - k_3 \left(\frac{k_1}{k_2}\right) a^2 = k_1 \frac{da}{d\tau}$$

$$\Rightarrow \frac{da}{d\tau} = 1 - a - \frac{k_3 k_1}{k_2^2} a^2,$$

$$= 1 - a - ka^2,$$

where $k = \frac{k_3 k_1}{k_2^2}$.