

MTH 113
Fall 2026
Exam 1
09/18/26

Name (Print):

key

This exam contains 9 pages (including this cover page) and 9 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

The following rules apply:

- **If you use a “fundamental theorem” you must indicate this** and explain why the theorem may be applied.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Short answer questions:** Questions labeled as “**Short Answer**” can be answered by simply writing an equation or a sentence or appropriately drawing a figure. No calculations are necessary or expected for these problems.
- **Unless the question is specified as short answer, mysterious or unsupported answers might not receive full credit.** An incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- Some problems will be used to assess mastery in either a pre-calculus or calculus outcome. In these problems, the particular learning outcome will be indicated in bold. **After grading the exam, the instructor will indicate mastery by circling the outcome and writing a capital M.**

Problem	Points	Score
1	15	
2	15	
3	10	
4	10	
5	15	
6	10	
7	10	
8	5	
9	10	
Total:	100	

Do not write in the table to the right.

1. (15 points) **Short Answer:** The figure below shows five vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ and $\vec{e}$ lying in two-dimensional space satisfying $\|\vec{a}\| = \|\vec{b}\| = 4$, $\|\vec{c}\| = 3$, and $\|\vec{d}\| = \|\vec{e}\| = 5$. Are the following statements true? Circle any true statements and cross out any false statements.

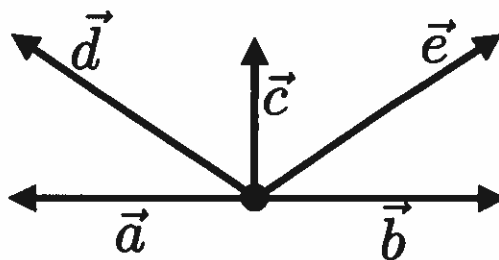
(a) $\vec{a} = -\vec{b}$

(b) ~~$\vec{e} = -\vec{d}$~~

(c) ~~$\vec{d} + \vec{e} = \vec{c}$~~

(d) $\vec{c} + \vec{b} = \vec{e}$

(e) ~~$\vec{c} - \vec{a} = \vec{d}$~~



2. (15 points) **Short Answer:** Determine if the following statement is correct (C) or incorrect (I). Just circle C or I. No need to show any work. In order for a statement to be correct it must be true in all cases.

C ☒ I If $x = f(t)$ and $y = g(t)$ are twice differentiable then $\frac{d^2y}{dx^2} = \frac{\frac{d^2y}{dt^2}}{\frac{d^2x}{dt^2}}$.

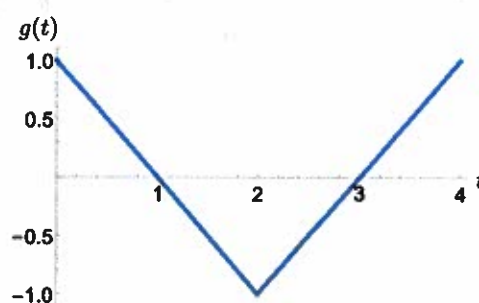
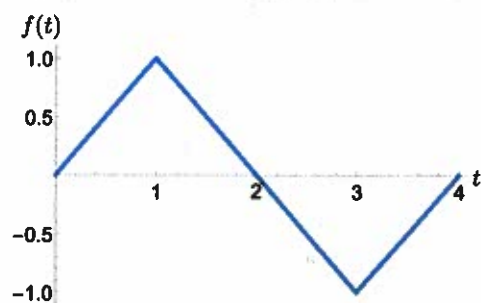
C ☒ I If a point is represented by (x, y) in Cartesian coordinates (where $x \neq 0$) and (r, θ) in polar coordinates then $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

☒ C I The polar curve $r = 2$, the curve defined by $x^2 + y^2 = 4$, and the curve defined by the parametric equations $x = 2 \sin(3t)$ and $y = 2 \cos(3t)$ for $0 \leq t \leq 2\pi$, all have the same graph in the xy -plane.

C ☒ I For any vectors $\vec{u}$ and $\vec{v}$ in three dimensional space, $\|\vec{u} + \vec{v}\| = \|\vec{u}\| + \|\vec{v}\|$.

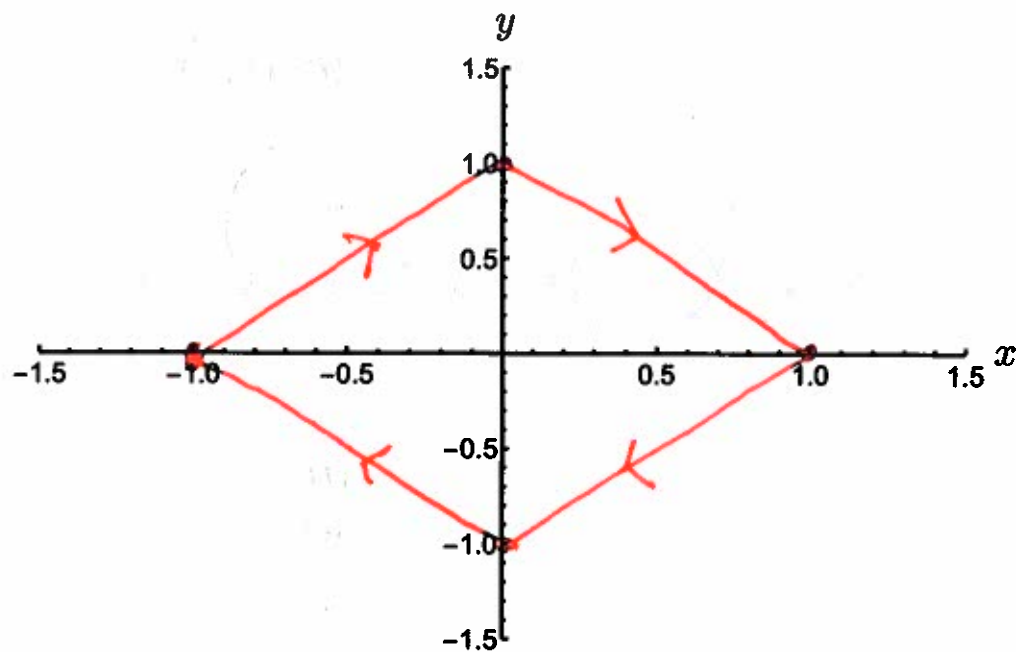
C ☒ I If the (x, y) coordinates of a particle at time t is given by the parametric equations $x(t) = 3t + 1$ and $y(t) = 2t^2 + 1$, then the speed of the particle at time $t = 3$ is the value of $\frac{dy}{dx}$ when $t = 3$.

3. (10 points) **Short Answer:** Below are the graphs of functions $f(t)$ and $g(t)$ over the interval $0 \leq t \leq 4$. On the axes provided below, sketch a graph of the parametric curve defined by

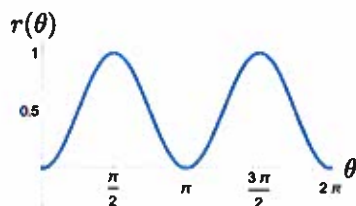


$$x(t) = f(t) \text{ and } y(t) = g(t), (0 \leq t \leq 4).$$

Be sure to label any x -intercepts and y -intercepts.



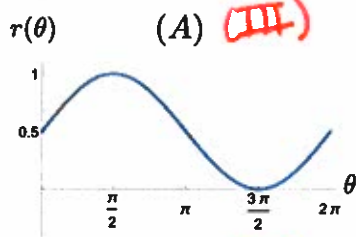
4. (10 points) **Short Answer:** In the figures below, the plots labeled (A)–(D) are graphs of the radial coordinate functions $r(\theta)$ on the interval $0 \leq \theta \leq 2\pi$. The plots labeled (I)–(IV), show the corresponding curves in the xy -plane. Match each radial coordinate function with its corresponding curve in the xy -plane.



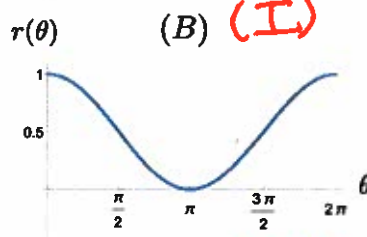
(A) (III)



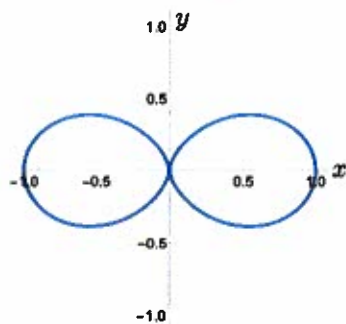
(B) (I)



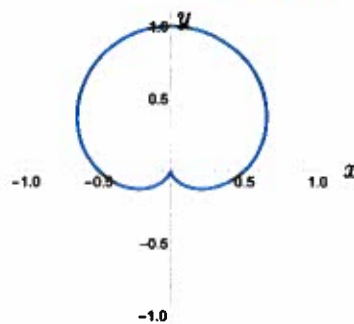
(C) (II)



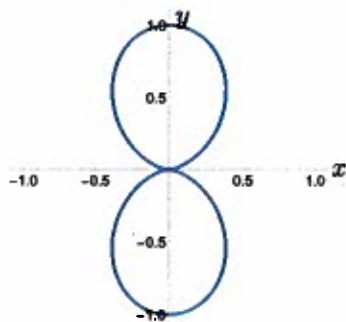
(D) (IV)



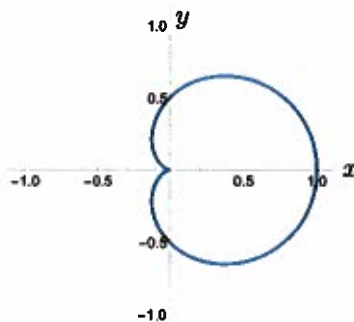
(I)



(II)



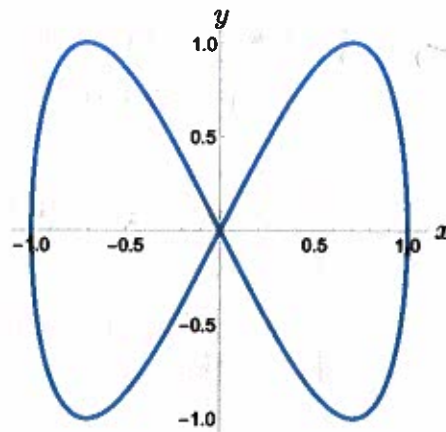
(III)



(IV)

5. (15 points) The figure below is the graph of the following parametric curve:

$$x(t) = \cos(2t), y(t) = \sin(4t), 0 \leq t \leq 2\pi.$$



- (a) (5 points) What is the exact value of the coordinate *in the first quadrant* at which the tangent to this curve is horizontal?

$$\frac{dy}{dx} = 4\cos(4t)$$

$$\text{When } t = \pi/8 \Rightarrow x = 1/\sqrt{2}, y = 1$$

- (b) (5 points) For what values of t ~~in the~~ does $x(t) = 0$ and $y(t) = 0$.

$$x(t) = 0 \text{ when } t = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4$$

$$y(t) = 0 \text{ when } t = 0, \pi/4, \pi/2, 3\pi/4, \pi, 5\pi/4, 3\pi/2, 7\pi/4, 2\pi$$

$$\text{They are both 0 when } t = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4$$

- (c) (5 points) Find the equation of the two tangent lines to this curve at the origin.

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{4\cos(4t)}{-2\sin(2t)}$$

$$\text{The two tangent lines are } y = 2x \text{ and } y = -2x.$$

$$\left. \frac{dy}{dx} \right|_{t=\pi/4} = 2$$

$$\left. \frac{dy}{dx} \right|_{t=3\pi/4} = -2$$

6. (10 points) Let $r(\theta) = f(\theta)$ denote a curve represented in polar coordinates. Show that on this curve:

$$\frac{dy}{dx} = \frac{\frac{df}{d\theta} \sin(\theta) + f(\theta) \cos(\theta)}{\frac{df}{d\theta} \cos(\theta) - f(\theta) \sin(\theta)}.$$

$$y(\theta) = f(\theta) \sin(\theta)$$

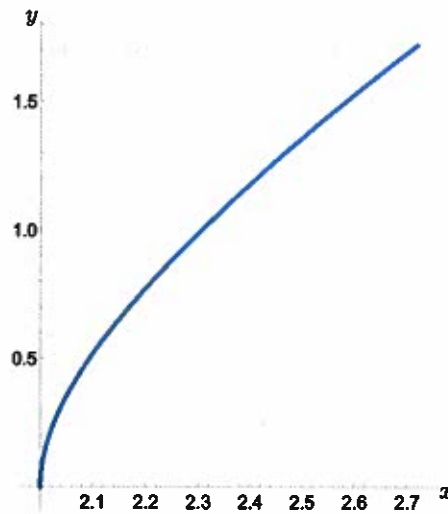
$$x(\theta) = f(\theta) \cos(\theta)$$

$$\Rightarrow \frac{dy}{d\theta} = \frac{df}{d\theta} \sin \theta + f(\theta) \cos(\theta)$$

$$\frac{dx}{d\theta} = \frac{df}{d\theta} \cos \theta - f(\theta) \sin(\theta)$$

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{df}{d\theta} \sin \theta + f(\theta) \cos(\theta)}{\frac{df}{d\theta} \cos \theta - f(\theta) \sin(\theta)}.$$

7. (10 points) Consider the parametric curve $x(t) = 1 + e^t - t$, $y(t) = e^t - 1$, $0 \leq t \leq 1$ graphed below. Find the area between the graph of this function and the x -axis.



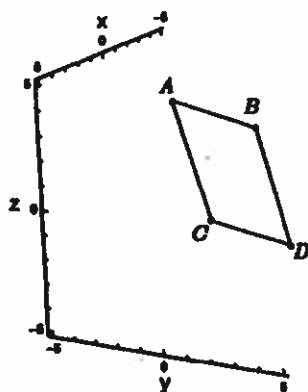
$$\text{Area} = \int_0^e y(x) dx = \int_0^1 y(t) \frac{dx}{dt} dt = \int_0^1 (e^t - 1)(e^t - 1) dt$$

$$\begin{aligned} \Rightarrow \text{Area} &= \int_0^1 (e^{2t} - 2e^t + 1) dt = \left(\frac{1}{2} e^{2t} - 2e^t + t \right) \Big|_0^1 \\ &= \frac{1}{2} e^2 - 2e + 1 - \frac{1}{2} + 2 \\ &= \frac{1}{2} e^2 - 2e + \frac{5}{2} \end{aligned}$$

8. (5 points) Let $a, b, c > 0$. Find an equation of the sphere that passes through the origin and whose center is (a, b, c) .

$$(x-a)^2 + (y-b)^2 + (z-c)^2 = a^2 + b^2 + c^2$$

9. (10 points) A parallelogram in three dimensional space has vertices located at $A = (2, -1, 3)$, $B = (1, 0, -1)$, $C = (1, 2, 3)$, and a fourth point D as illustrated in the figure below.



- (a) (5 points) Find the vectors $\vec{AB}$ and $\vec{AC}$.

$$\vec{AB} = \langle -1, 1, -4 \rangle$$

$$\vec{AC} = \langle -1, 3, 0 \rangle$$

- (b) (5 points) Find the coordinates of the fourth point D .

We know $\vec{BD} = \vec{AC}$. If we let (x, y, z) denote the coordinates of D then

$$\vec{BD} = \langle x-1, y, z+1 \rangle = \langle -1, 3, 0 \rangle$$

$$\Rightarrow x=0, y=3, z=-1$$

$$\Rightarrow D = (0, 3, -1).$$

