Homework #2

pg. 680, #8

$x = \sqrt{t}$

$y = t^2 - 2t$

$\Rightarrow \frac{dy}{dt} = 2t - 2, \quad \frac{dx}{dt} = \frac{1}{2}t^{-1/2}$

The slope when $t=4$ is given by

$$m = \left.\frac{dy}{dx}\right|_{t=4} = \left.\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right|_{t=4} = \frac{6}{1/4} = 24.$$

At $t=4$, we have that $x=2$ and $y=8$. Therefore, the equation of the tangent line is

$$\begin{cases} x = \frac{1}{4}(t-4)+2 \\ y = 6(t-4)+8 \end{cases} \quad \text{or} \quad y = 24(x-2)+8$$

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pg. 680, #24

$x = e^{\sin\theta}$

$y = e^{\cos\theta}$

$\Rightarrow \frac{dx}{d\theta} = \cos\theta\, e^{\sin\theta}, \quad \frac{dy}{d\theta} = -\sin\theta\, e^{\cos\theta}$

The tangent line will be vertical when $\frac{dx}{d\theta} = 0$ which occurs when $\theta = \pi/2 + n\pi$ $(n \in \mathbb{Z})$ which coorespends to the following Cartesian coordinates: $(e, 1), (e^{-1}, 1)$.

The tangent line will be horizontal when $\frac{dy}{d\theta} = 0$ which occurs when $\theta = n\pi$ $(n \in \mathbb{Z})$ which corresponds to the following Cartesian coordinates: $(1, e), (1, e^{-1})$

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pg. 681, #36

$$A=\int_0^1 y(x)\,dx$$

However,

$$y=\sin(t)\cos(t),\quad x=\sin(t)$$

$$\Rightarrow dx=\cos(t)\,dt$$

Therefore,

$$A=\int_0^{\pi/2}\sin^2(t)\cos(t)\,dt$$

Let $u=\sin(t)\Rightarrow du=\cos(t)\,dt$

$$\Rightarrow A=\int_0^1 u^2\,du=\tfrac{1}{3}.$$

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$$x=e^t-t,\quad y=4e^{t/2}$$

$$\Rightarrow \frac{dx}{dt}=e^t-1,\quad \frac{dy}{dt}=2e^{t/2}$$

$$\Rightarrow L=\int_0^2\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$$

$$=\int_0^2\left((e^t-1)^2+4e^t\right)^{1/2}dt$$

$$=\int_0^2\left(e^{2t}-2e^t+1+4e^t\right)^{1/2}dt$$

$$=\int_0^2\left(e^{2t}+2e^t+1\right)^{1/2}dt$$

$$=\int_0^2\left((e^t+1)^2\right)^{1/2}dt$$

$$=\int_0^2(e^t+1)\,dt$$

$$=e^2+2-1$$

$$=2+e^2$$

pg. 682, #62

$x = a\sin\theta$

$y = b\cos\theta$

$\Rightarrow \frac{dx}{d\theta} = a\cos\theta, \ \frac{dy}{d\theta} = -b\sin\theta$

$\Rightarrow L = \int_0^{2\pi} \sqrt{a^2\cos^2\theta + b^2\sin^2\theta}\, d\theta$

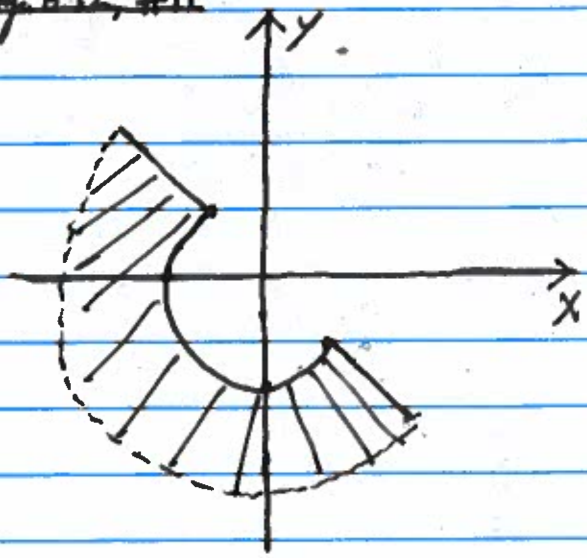
$= \int_0^{2\pi} \sqrt{a^2\cos^2\theta + a^2\sin^2\theta + b^2\sin^2\theta - a^2\sin^2\theta}\, d\theta$

$= \int_0^{2\pi} \sqrt{a^2 - (a^2-b^2)\sin^2\theta}\, d\theta$

$= a\int_0^{2\pi} \sqrt{1 - \frac{(a^2-b^2)}{a^2}\sin^2\theta}\, d\theta$

$= a\int_0^{2\pi} \sqrt{1 - e^2\sin^2\theta}\, d\theta.$

pg. 692, #11



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$y = -2x^2$

$\Rightarrow r\sin\theta = -2r^2\cos^2(\theta)$

$\Rightarrow \sin\theta + 2r\cos^2\theta = 0$

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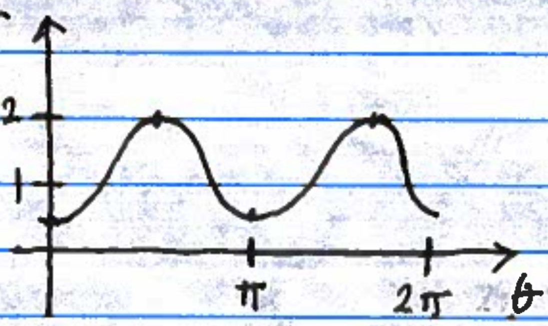

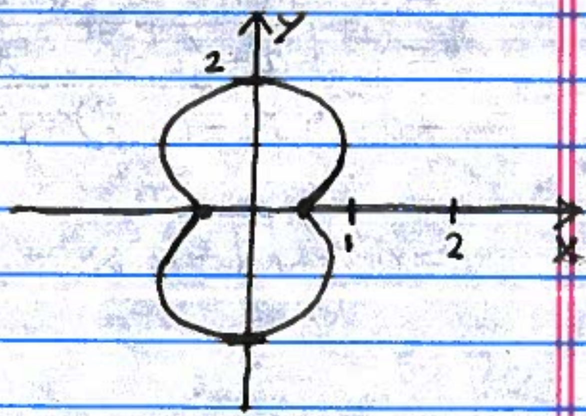

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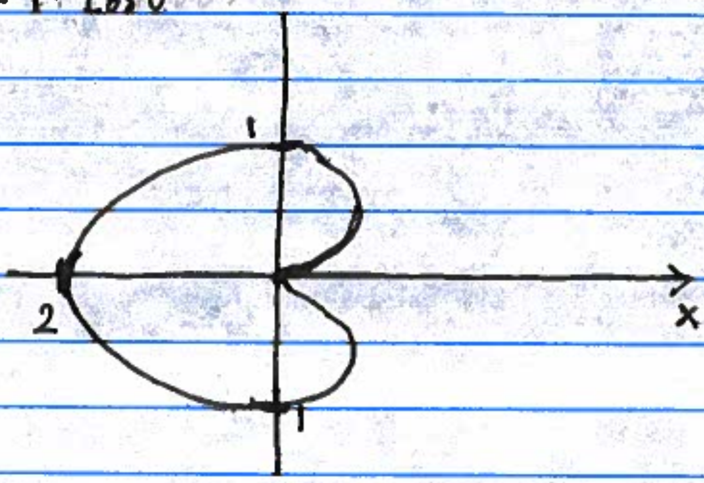
$r = 1 - \cos\theta$



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