

Homework #3

pg 700, #6

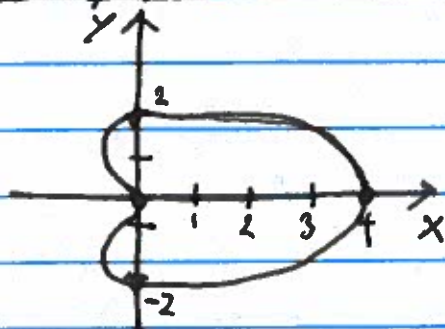
$$A = \int_{\pi/2}^{\pi} \frac{1}{2} r(\theta)^2 d\theta = \frac{1}{2} \int_{\pi/2}^{\pi} (2 + \cos\theta)^2 d\theta = \frac{1}{2} \int_{\pi/2}^{\pi} (4 + 4\cos\theta + \cos^2\theta) d\theta$$

$$\Rightarrow A = \frac{1}{2} \int_{\pi/2}^{\pi} (4 + 4\cos\theta + \frac{1+\cos 2\theta}{2}) d\theta$$

$$\Rightarrow A = \pi + 2\sin\theta \Big|_{\pi/2}^{\pi} + \frac{\pi}{8} + \frac{1}{4}\sin(2\theta) \Big|_{\pi/2}^{\pi}$$

$$\Rightarrow A = \frac{9\pi}{8} - 2$$

pg 700, #10



$$A = \int_0^{2\pi} \frac{1}{2} r(\theta)^2 d\theta = \int_0^{2\pi} \frac{1}{2} (2 + 2\cos\theta)^2 d\theta = 2 \int_0^{2\pi} (1 + 2\cos\theta + \cos^2\theta) d\theta$$

$$\Rightarrow A = 4\pi + 2\pi = 6\pi$$

pg 700, #40

$$r = \cos\theta, r = \sin 2\theta$$

$$\Rightarrow \cos\theta = \sin 2\theta$$

$$\Rightarrow \cos\theta = 2\sin\theta\cos\theta$$

$$\Rightarrow \cos\theta(1 - 2\sin\theta) = 0$$

$$\Rightarrow \cos\theta = 0 \text{ or } \sin\theta = \frac{1}{2}$$

$$\Rightarrow \theta = \pi/2, 3\pi/2 \text{ or } \theta = \pi/6, 5\pi/6$$

The points of intersection are

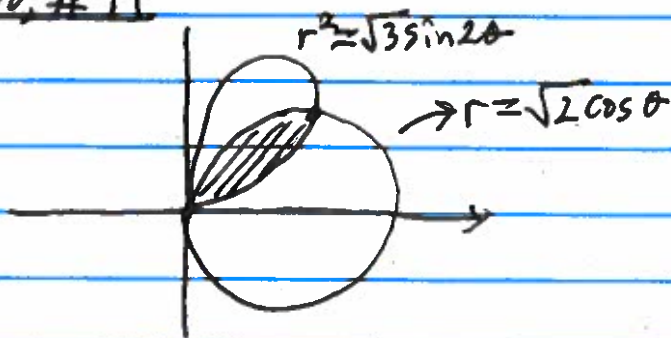
$$\theta = \pi/2, r = 0$$

$$\theta = 3\pi/2, r = 0$$

$$\theta = \pi/6, r = \sqrt{3}/2$$

$$\theta = 5\pi/6, r = -\sqrt{3}/2$$

Pr. 700, #44



$$r^2 = \sqrt{3} \sin(2\theta) = 2 \cos^2 \theta$$

$$\Rightarrow \sqrt{3} \cdot 2 \sin \theta \cos \theta = 2 \cos^2 \theta$$

$$\Rightarrow \sqrt{3} \cdot 2 \sin \theta \cos \theta - 2 \cos^2 \theta = 0$$

$$\Rightarrow 2 \cos \theta (\sqrt{3} \sin \theta - \cos \theta) = 0$$

$$\Rightarrow 2 \cos \theta = 0 \text{ or } \sqrt{3} \sin \theta - \cos \theta = 0$$

$$\Rightarrow 2 \cos \theta = 0 \text{ or } \tan \theta = \frac{1}{\sqrt{3}}$$

$$A = \int_0^{\pi/6} \frac{\sqrt{3}}{2} \sin(2\theta) d\theta + \int_{\pi/6}^{\pi/2} \cos^2 \theta d\theta$$

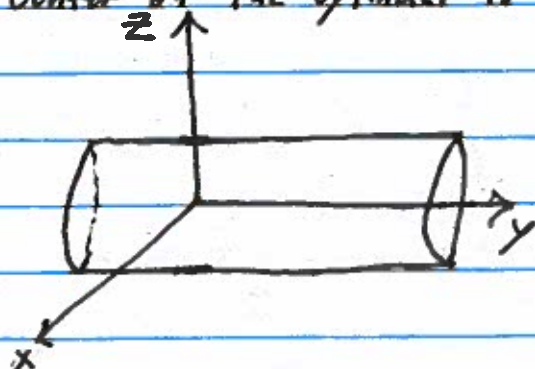
$$= \frac{\sqrt{3}}{4} \cos(2\theta) \Big|_0^{\pi/6} + \int_{\pi/6}^{\pi/2} \frac{1 + \cos(2\theta)}{2} d\theta$$

$$= \frac{\sqrt{3}}{4} (\cos(\pi/3) - 1) + \frac{\pi}{2} - \frac{\pi}{6} - \frac{1}{4} \cos(\pi) + \frac{1}{4} \cos(\frac{\pi}{3})$$

pg. 835, #8

$$x^2 + z^2 = 9$$

This is a cylinder oriented in the y -direction of radius 3.
The center of the cylinder is the y -axis.



pg. 835, #18

The distance from the origin to the point $(1, 2, 3)$ is $\sqrt{14}$.

The equation for the sphere is therefore:

$$(x-1)^2 + (y-2)^2 + (z-3)^2 = 14.$$

pg. 835, #42

$$x^2 + y^2 + z^2 > 2z$$

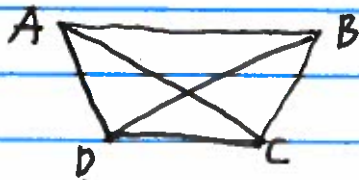
$$\Rightarrow x^2 + y^2 + z^2 - 2z > 0$$

$$\Rightarrow x^2 + y^2 + z^2 - 2z + 1 - 1 > 0$$

$$\Rightarrow x^2 + y^2 + (z-1)^2 > 1$$

This inequality corresponds to the points lying outside a sphere of radius 1 centered at $(0, 0, 1)$.

pg. 844, #4



(a) $\vec{AB} + \vec{BC} = \vec{AC}$

(b) $\vec{CD} + \vec{DB} = \vec{CB}$

(c) $\vec{DB} - \vec{AB} = \vec{DA}$

(d) $\vec{DC} + \vec{CA} + \vec{AB} = \vec{DB}$

pg. 844, #20

$$\vec{a} = 5\vec{i} + 3\vec{j} = \langle 5, 3 \rangle$$

$$\vec{b} = -\vec{i} - 2\vec{j} = \langle -1, -2 \rangle$$

(i) $\vec{a} + \vec{b} = \langle 4, 1 \rangle = 4\vec{i} + \vec{j}$

(ii) $4\vec{a} + 2\vec{b} = \langle 20, 12 \rangle + \langle -2, -4 \rangle = \langle 18, 8 \rangle = 18\vec{i} + 8\vec{j}$

(iii) $\|\vec{a}\| = \sqrt{5^2 + 3^2} = \sqrt{34}$

(iv) $\|\vec{a} - \vec{b}\| = \|\langle 6, 5 \rangle\| = \sqrt{36 + 25} = \sqrt{61}$

pg. 844, #42

$$y = 2\sin(x)$$

$$\Rightarrow \frac{dy}{dx} = 2\cos(x)$$

$$\Rightarrow \left. \frac{dy}{dx} \right|_{\pi/6} = \frac{2 \cdot \sqrt{3}}{2} = \sqrt{3}$$

a) Unit tangent vector $\frac{\langle 1, \sqrt{3} \rangle}{\|\langle 1, \sqrt{3} \rangle\|} = \langle \frac{1}{2}, \frac{\sqrt{3}}{2} \rangle$

b) Unit normal vector $= \langle -\frac{\sqrt{3}}{2}, \frac{1}{2} \rangle$