

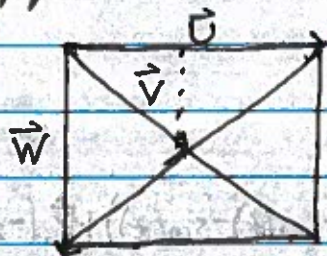
Homework #4

pg. 852, #6

$$\vec{a} = \langle p, -p, 2p \rangle, \vec{b} = \langle 2q, q, -q \rangle$$

$$\Rightarrow \vec{a} \cdot \vec{b} = 2pq - pq - 2pq = -pq$$

pg. 853, #12



By construction $(\frac{1}{2})^2 + (\frac{1}{2})^2 = \|\vec{v}\|^2$ and thus $\|\vec{v}\| = \frac{1}{\sqrt{2}}$. By construction we also have that

$$\vec{v} \cdot \vec{w} = 0 \text{ and } \vec{v} \cdot \vec{u} = \|\vec{v}\| \cdot \|\vec{u}\| \cos(74^\circ) = \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = \frac{1}{2}$$

pg. 854, #54

If $(\vec{r}-\vec{a}) \cdot (\vec{r}-\vec{b}) = 0$ then

$$\vec{r} \cdot \vec{r} - \vec{a} \cdot \vec{r} - \vec{b} \cdot \vec{r} + \vec{a} \cdot \vec{b} = 0$$

$$\Rightarrow \vec{r} \cdot \vec{r} - (\vec{a} + \vec{b}) \cdot \vec{r} + \vec{a} \cdot \vec{b} = 0$$

$$\Rightarrow \vec{r} \cdot \vec{r} - (\vec{a} + \vec{b}) \cdot \vec{r} + \frac{(\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b})}{4} - \frac{(\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b})}{4} + \vec{a} \cdot \vec{b} = 0$$

$$\Rightarrow \left(\vec{r} - \frac{(\vec{a} + \vec{b})}{2} \right) \cdot \left(\vec{r} - \frac{(\vec{a} + \vec{b})}{2} \right) - \frac{\vec{a} \cdot \vec{a} + 2\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{b}}{4} + \vec{a} \cdot \vec{b} = 0$$

$$\Rightarrow \left\| \vec{r} - \frac{(\vec{a} + \vec{b})}{2} \right\|^2 = \frac{\vec{a} \cdot \vec{a} - 2\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{b}}{4}$$

$$\Rightarrow \left\| \vec{r} - \frac{(\vec{a} + \vec{b})}{2} \right\|^2 = \left\| \frac{\vec{a} - \vec{b}}{2} \right\|^2$$

This is the equation of a sphere of radius $\left\| \frac{\vec{a} - \vec{b}}{2} \right\|$ centered at $(x, y, z) = \left(\frac{a_1 + b_1}{2}, \frac{a_2 + b_2}{2}, \frac{a_3 + b_3}{2} \right)$.

pg. 854, #64

If $\vec{u} + \vec{v}$ and $\vec{u} - \vec{v}$ are orthogonal then

$$(\vec{u} + \vec{v}) \cdot (\vec{u} - \vec{v}) = \vec{u} \cdot \vec{u} + \vec{v} \cdot \vec{u} - \vec{u} \cdot \vec{v} - \vec{v} \cdot \vec{v} = 0$$

$$\Rightarrow \vec{u} \cdot \vec{u} = \vec{v} \cdot \vec{v}$$

$$\Rightarrow \|\vec{u}\|^2 = \|\vec{v}\|^2$$

$$\Rightarrow \|\vec{u}\| = \|\vec{v}\|$$

pg. 861, #6

$$\vec{a} = t\vec{i} + \cos(t)\vec{j} + \sin(t)\vec{k}$$

$$\vec{b} = \vec{i} - \sin(t)\vec{j} + \cos(t)\vec{k}$$

$$\Rightarrow \vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ t & \cos(t) & \sin(t) \\ 1 & -\sin(t) & \cos(t) \end{vmatrix} = \vec{i}(\cos^2(t) + \sin^2(t)) - \vec{j}(t\cos(t) - \sin(t)) + \vec{k}(-t\sin(t) - \cos(t))$$

$$\Rightarrow \vec{a} \times \vec{b} = \vec{i} - (t\cos(t) - \sin(t))\vec{j} - (t\sin(t) + \cos(t))\vec{k}$$

pg. 862, #20

$$(\vec{j} - \vec{k}) \times (\vec{i} + \vec{j}) = \vec{j} \times \vec{i} - \vec{k} \times \vec{i} + \vec{j} \times \vec{j} - \vec{k} \times \vec{j} = -\vec{k} + \vec{j} + \vec{i}$$

Therefore, two unit vectors orthogonal to $\vec{j} - \vec{k}$ and $\vec{i} + \vec{j}$ are given by

$$\hat{n}_1 = \frac{1}{\sqrt{3}}(\vec{i} + \vec{j} - \vec{k}) \text{ and } \hat{n}_2 = \frac{1}{\sqrt{3}}(\vec{i} + \vec{j} - \vec{k})$$

pg. 862, #49

$$(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b}) = \vec{a} \times \vec{a} + \vec{a} \times \vec{b} - \vec{b} \times \vec{a} - \vec{b} \times \vec{b} = \vec{a} \times \vec{b} + \vec{a} \times \vec{b} = 2(\vec{a} \times \vec{b})$$

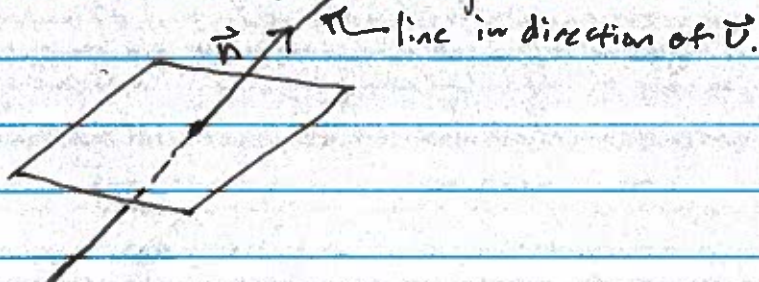
pg. 872, #14

$$\text{Let } \vec{v}_1 = \langle 1-(-2), 1-4, 1-0 \rangle = \langle 3, -3, 1 \rangle \text{ and } \vec{v}_2 = \langle 3-2, -1-3, -8-4 \rangle = \langle 1, -4, -12 \rangle$$

Consequently, $\vec{v}_1 \cdot \vec{v}_2 = 3 + 12 - 12 = 3$ and thus the lines are not orthogonal.

pg. 872, #26.

A vector in the direction of the line $x=1-8t, y=-1-7t, z=4+2t$ is given by $\vec{v} = \langle -8, -7, 2 \rangle$. If we let $\vec{n}$ denote the normal to the plane we have that for the plane to be orthogonal to this line we require that $\vec{n}$ and $\vec{v}$ are parallel.



So the equation is given by:
 $-8x - 7y + 2z = 0$.

pg. 872, #34

From the points we can compute tangent vectors in the plane

$$\vec{u} = \langle -2-3, -2-0, 3-(-1) \rangle = \langle -5, -2, 4 \rangle$$

$$\vec{v} = \langle 7-3, 1-0, -4-(-1) \rangle = \langle 4, 1, -3 \rangle$$

$$\Rightarrow \vec{u} \times \vec{v} = \langle -5, -2, 4 \rangle \times \langle 4, 1, -3 \rangle$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -5 & -2 & 4 \\ 4 & 1 & -3 \end{vmatrix} = \vec{i}(6-4) - \vec{j}(15-16) + \vec{k}(-5+8)$$

$$\Rightarrow \vec{u} \times \vec{v} = 2\vec{i} + \vec{j} + 3\vec{k}$$

So one possible equation is given by

$$2(x-3) + y + 3(z+1) = 0$$