Lecture #1: Parametric Curves

Motivating Idea:

Calc 1 and 2:

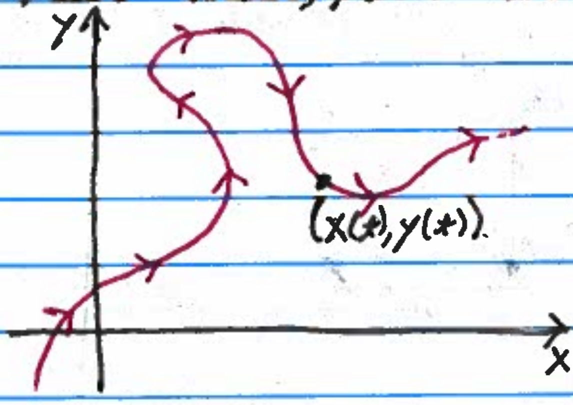
$f: \mathbb{R} \to \mathbb{R}$ ← outputs # (Range)

↑ takes in # (Domain)

Parametric Curve:

$F: \mathbb{R} \to \mathbb{R}^2$ ← outputs two numbers (coordinates)

$\Rightarrow F(t) = (x(t), y(t)) = (f(t), g(t))$



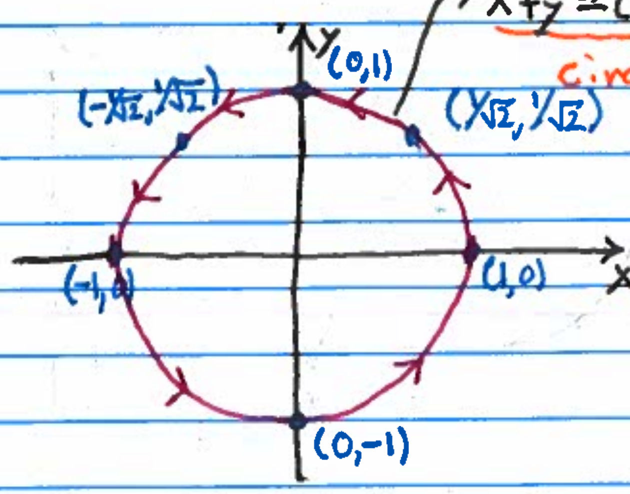
Example:

$F(t) = (\cos(t), \sin(t))$

$\Rightarrow x(t) = \cos(t)$ → parameter = time

$y(t) = \sin(t)$

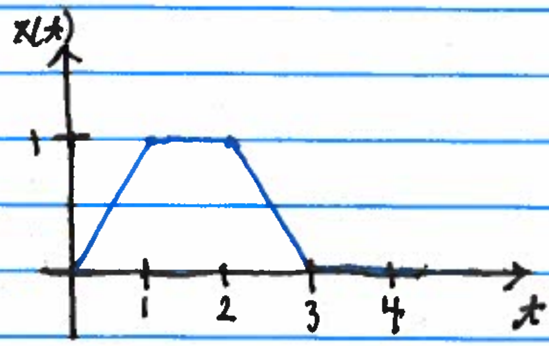
t	x	y
0	1	0
$\pi/4$	$1/\sqrt{2}$	$1/\sqrt{2}$
$\pi/2$	0	1
$3\pi/4$	$-1/\sqrt{2}$	$1/\sqrt{2}$
π	-1	0
$3\pi/2$	0	-1

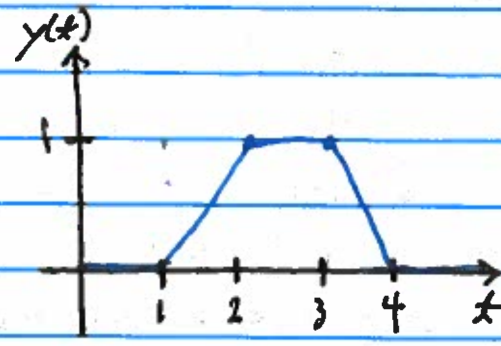


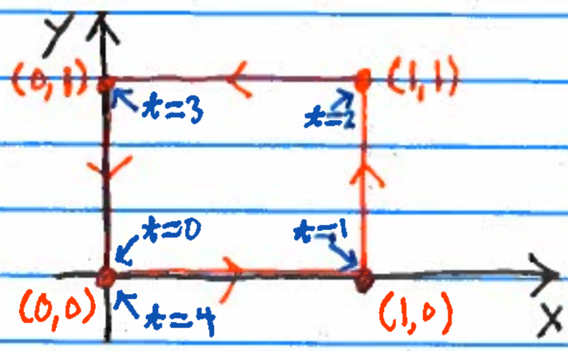
$\underbrace{x^2 + y^2 = \cos^2(t) + \sin^2(t) = 1}_{\text{circle of radius 1}}$

Example:

Describe the motion of the particle whose coordinates are plotted below



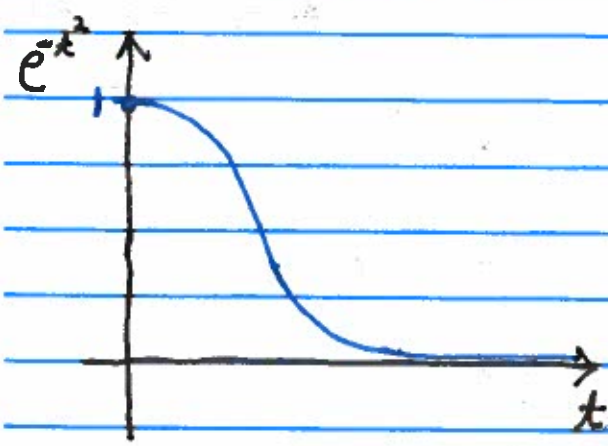


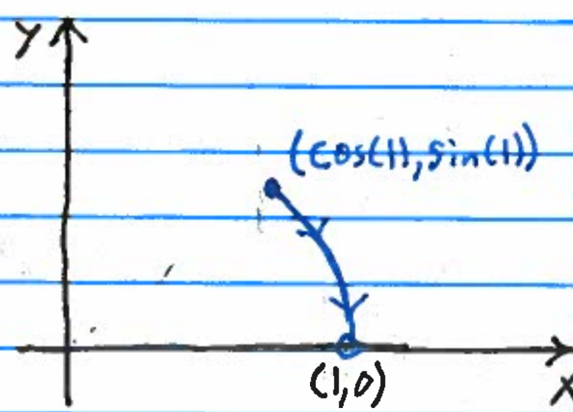


Example:

Describe the motion of the particle whose x,y coordinates are given by:

$$x(t) = \cos(e^{-t^2}), \quad t \geq 0$$
$$y(t) = \sin(e^{-t^2})$$





Example:

The path of a particle is given by

$$x = t^2,\ y = t$$

Sketch the path of the particle.

$$t = \pm\sqrt{x}$$

$$\Rightarrow y = \pm\sqrt{x}$$ (Cartesian Representation)