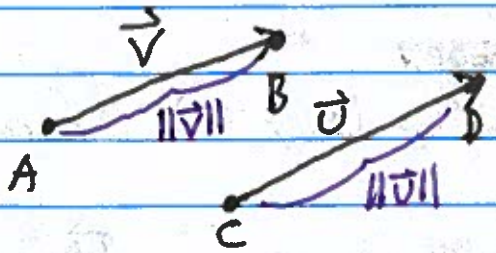
Lecture #5: Vectors

Geometric Description:

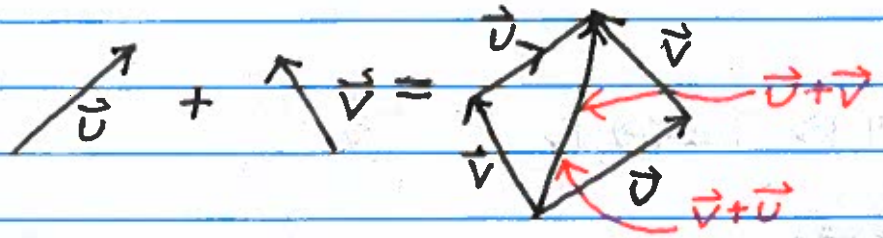


$\vec{v}, \vec{u}$ are vectors which are line segments with arrowheads indicating direction.

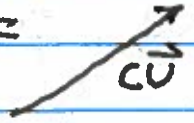
* In this case $\vec{u}, \vec{v}$ have the same length and direction so we say $\vec{u} = \vec{v}$

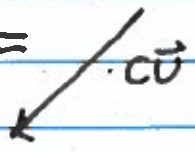
* The magnitude of a vector is its length and is denoted
$\|\vec{v}\|$ = length of $\vec{v}$

Adding:

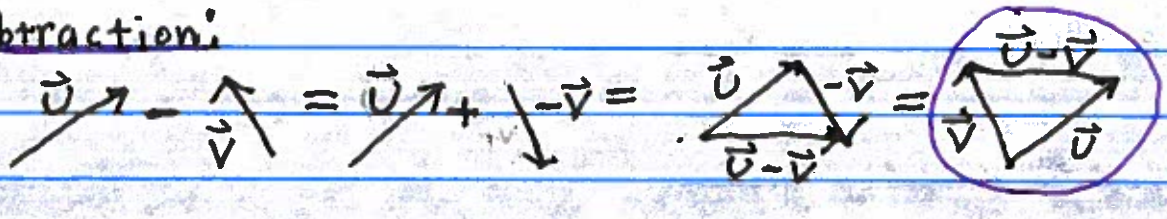


Scalar Multiplication:

(i) $c > 0$: $c \cdot \vec{u} =$ 

(ii) $c < 0$: $c \cdot \vec{u} =$ 

Subtraction:



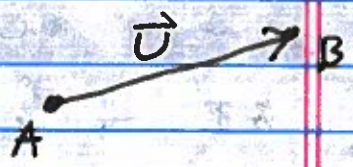
hard to remember.

Algebraic Description

Given points $A=(x_1, y_1, z_1)$ and $B=(x_2, y_2, z_2)$ the coordinate representation of

$$\vec{u}=\overrightarrow{AB}=\langle x_2-x_1, y_2-y_1, z_2-z_1\rangle$$

↑ displacement vector



Rules:

1. If $\vec{u}=\langle u_1, u_2, u_3\rangle$ and $\vec{v}=\langle v_1, v_2, v_3\rangle$
   $\Rightarrow \vec{u}+\vec{v}=\langle u_1+v_1, u_2+v_2, u_3+v_3\rangle$
2. If $\vec{u}=\langle u_1, u_2, u_3\rangle$ then $c\vec{u}=\langle cu_1, cu_2, cu_3\rangle$.

Example:

If $\vec{a}=\langle 4,3\rangle$, $\vec{b}=\langle -2,1\rangle$

- $\vec{a}+\vec{b}=\langle 2,4\rangle$
- $\|\vec{a}\|=\sqrt{16+9}=5$
- $\vec{a}-\vec{b}=\langle 6,2\rangle$
- $2\vec{a}+5\vec{b}=2\cdot\langle 4,3\rangle+5\langle -2,1\rangle=\langle 8,6\rangle+\langle -10,5\rangle=\langle -2,11\rangle$.

Unit Vectors

A vector of length 1 is called a unit vector. The standard unit vectors are

$$\vec{i} = \langle 1,0,0\rangle = \vec{e}_1$$
$$\vec{j} = \langle 0,1,0\rangle = \vec{e}_2$$
$$\vec{k} = \langle 0,0,1\rangle = \vec{e}_3$$

Note, if

$$\vec{v} = \langle v_1, v_2, v_3\rangle$$

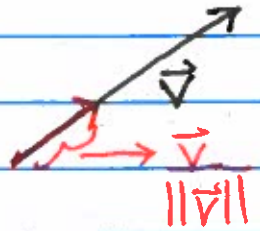
then

$$\vec{v} = v_1\vec{i} + \langle 0, v_2, v_3\rangle$$
$$= v_1\vec{i} + v_2\vec{j} + \langle 0,0,v_3\rangle$$
$$= v_1\vec{i} + v_2\vec{j} + v_3\vec{k}.$$

The direction of a vector $\vec{v}$ is

$$\frac{\vec{v}}{\|\vec{v}\|}$$

it is a unit vector pointing in the direction of $\vec{v}$.



Check:

$$\left\| \frac{\vec{v}}{\|\vec{v}\|} \right\| = \left\| \left\langle \frac{v_1}{(v_1^2+v_2^2)^{1/2}}, \frac{v_2}{(v_1^2+v_2^2)^{1/2}} \right\rangle \right\|$$
$$= \left(\frac{v_1^2}{v_1^2+v_2^2} + \frac{v_2^2}{v_1^2+v_2^2} \right)^{1/2} = \left(\frac{v_1^2+v_2^2}{v_1^2+v_2^2} \right)^{1/2} = 1.$$