

Lecture #7: Functions of Several Variables

$$F: \mathbb{R}^2 \rightarrow \mathbb{R}$$

↑
input pair
of numbers

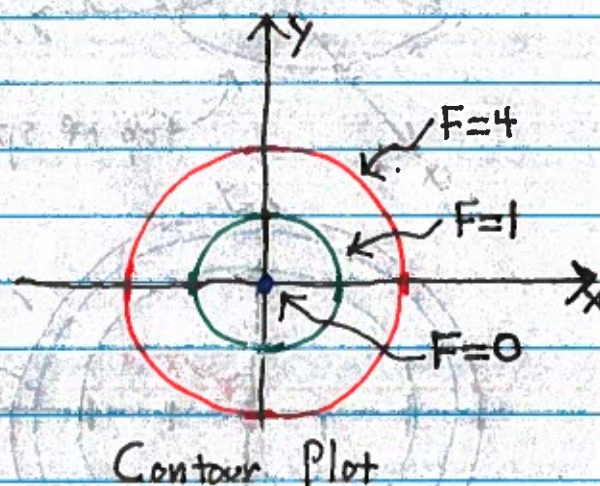
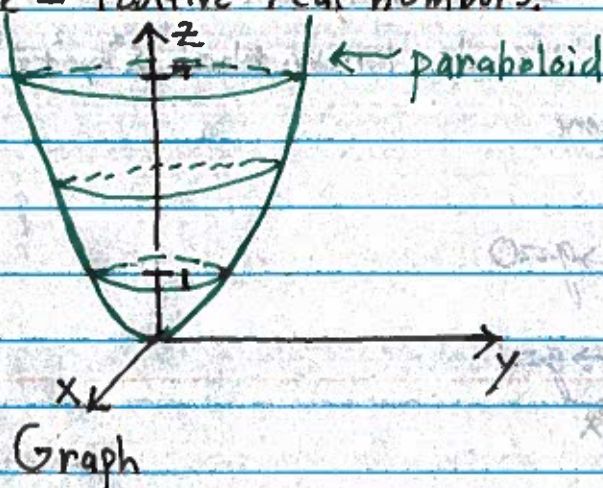
↖
output one number

Example:

$$F(x, y) = x^2 + y^2$$

Domain = All pairs of real numbers, i.e., $\mathbb{R}^2$.

Range = Positive real numbers.

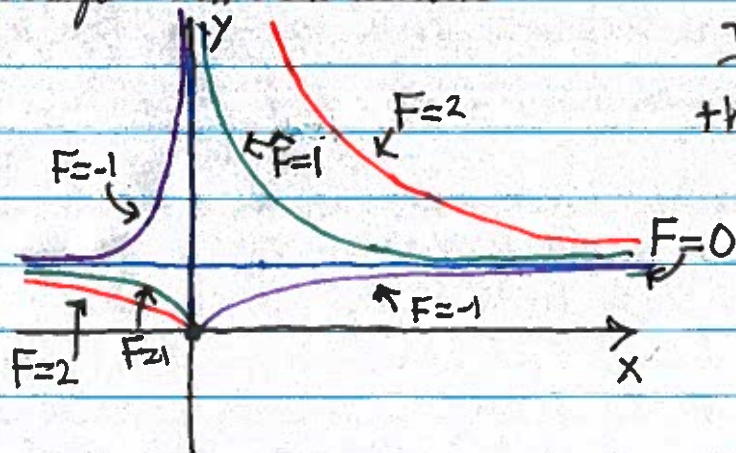


Example:

$$F(x, y) = x \ln(y)$$

Domain = $\{(x, y) \in \mathbb{R}^2 : y > 0\}$

Range = All real numbers



If we let $z = x \ln(y)$,
then if $x \neq 0$ we have
 $y = e^{z/x}$

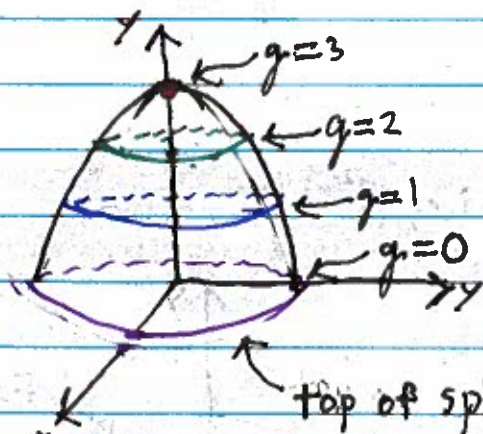
Contour Plot

Example:

$$g(x,y) = \sqrt{9-x^2-y^2}$$

$$\text{Domain: } x^2+y^2 \leq 9$$

$$\text{Range: } [0,3]$$



$$z=3 \Rightarrow \sqrt{9-x^2-y^2}=3 \Rightarrow x^2+y^2=0$$

$$z=2 \Rightarrow \sqrt{9-x^2-y^2}=2 \Rightarrow x^2+y^2=5$$

$$z=1 \Rightarrow x^2+y^2=8$$

$$z=0 \Rightarrow x^2+y^2=9$$

We also know that $x^2+y^2+z^2=9$

