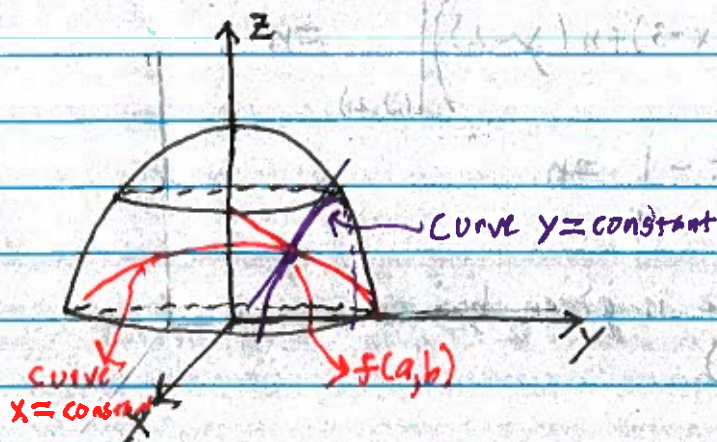


Lecture #9: Partial Derivatives



$$f_x(a, b) = \left. \frac{\partial f}{\partial x} \right|_{(a, b)} = \lim_{\Delta x \rightarrow 0} \frac{f(a + \Delta x, b) - f(a, b)}{\Delta x}, \quad \left(\begin{array}{l} \text{slope of tangent} \\ \text{line in } x\text{-direction} \end{array} \right)$$

$$f_y(a, b) = \left. \frac{\partial f}{\partial y} \right|_{(a, b)} = \lim_{\Delta y \rightarrow 0} \frac{f(a, b + \Delta y) - f(a, b)}{\Delta y}, \quad \left(\begin{array}{l} \text{slope of tangent} \\ \text{line in } y\text{-direction} \end{array} \right)$$

Example:

$$\text{Let } f(x, y) = \frac{x^2}{y+1}$$

$$\Rightarrow \frac{\partial f}{\partial x} = \frac{\partial}{\partial x} \left(\frac{x^2}{y+1} \right) = \frac{1}{y+1} \frac{\partial x^2}{\partial x} = \frac{2x}{y+1}$$

$$\Rightarrow \frac{\partial f}{\partial y} = \frac{\partial}{\partial y} \left(\frac{x^2}{y+1} \right) = x^2 \frac{\partial}{\partial y} \left(\frac{1}{y+1} \right) = -\frac{x^2}{(y+1)^2}$$

Near (3, 2), $f(x, y)$ can be approximated by a plane:

$$z - z_0 = m(x - x_0) + n(y - y_0)$$

$$\Rightarrow z - 3 = m(x - 3) + n(y - 2)$$

$$\Rightarrow \left. \frac{\partial (z - 3)}{\partial x} \right|_{(3, 2)} = \left. \frac{\partial}{\partial x} (m(x - 3) + n(y - 2)) \right|_{(3, 2)} = m$$

$$\Rightarrow \left. \frac{\partial z}{\partial x} \right|_{(3, 2)} = \left. \frac{2x}{y+1} \right|_{(3, 2)} = 2 = m$$

We also have that:

$$\left. \frac{\partial (z-3)}{\partial y} \right|_{(3,2)} = \left. \frac{\partial (m(x-3)+n(y-2))}{\partial y} \right|_{(3,2)} = n$$

$$\Rightarrow \left. \frac{\partial z}{\partial y} \right|_{(3,2)} = \left. \frac{-x^2}{(y+1)^2} \right|_{(3,2)} = -\frac{1}{9} = n.$$

Therefore, the tangent plane is given by:

$$z-3 = 2(x-3) - \frac{1}{9}(y-2).$$

Examples:

1. $f(x,y) = y^2 e^{3x}$

$$\frac{\partial f}{\partial x} = \frac{\partial (y^2 e^{3x})}{\partial x} = y^2 \frac{\partial e^{3x}}{\partial x} = 3y^2 e^{3x}$$

$$\frac{\partial f}{\partial y} = \frac{\partial (y^2 e^{3x})}{\partial y} = e^{3x} \frac{\partial y^2}{\partial y} = 2y e^{3x}$$

2. $z = (3xy+2x)^5$

$$\frac{\partial z}{\partial x} = \frac{\partial (3xy+2x)^5}{\partial x} = 5(3xy+2x)^4 \left(\frac{\partial (3xy+2x)}{\partial x} \right) = 5(3xy+2x)^4 (3y+2)$$

$$\frac{\partial z}{\partial y} = \frac{\partial (3xy+2x)^5}{\partial y} = 5(3xy+2x)^4 \left(\frac{\partial (3xy+2x)}{\partial y} \right) = 5(3xy+2x)^4 3x$$

3. $g(x,y) = e^{x+3y} \sin(xy)$

$$\frac{\partial g}{\partial x} = \frac{\partial (e^{x+3y})}{\partial x} \sin(xy) + e^{x+3y} \frac{\partial (\sin(xy))}{\partial x}$$

$$= e^{x+3y} \sin(xy) + e^{x+3y} \cos(xy) y$$

$$\frac{\partial g}{\partial y} = \frac{\partial (e^{x+3y})}{\partial y} \sin(xy) + e^{x+3y} \frac{\partial (\sin(xy))}{\partial y}$$

$$= 3e^{x+3y} \sin(xy) + e^{x+3y} \cos(xy) x$$