

Quiz #1

MTH 383/683: Fall 2026

Instructor: John Gemmer

Problem 1. Consider the probability space $(\Omega, \mathcal{F}, P)$. For $A, B \in \mathcal{F}$, write down the definition of what it means for A and B to be independent.

$$P(A \cap B) = P(A)P(B)$$

Problem 2. Let X be a random variable on a probability space $(\Omega, \mathcal{F}, P)$ that is uniformly distributed on $[0, 1]$ and let $Y = X^3$. Find the CDF of Y .

$$P(Y \leq y) = \begin{cases} 0, & y \leq 0 \\ P(0 \leq Y \leq y), & 0 \leq y \leq 1 \\ 1, & y > 1 \end{cases}$$

If $0 \leq y \leq 1$ then

$$P(0 \leq Y \leq y) = P(0 \leq X^3 \leq y) = P(0 \leq X \leq y^{1/3})$$

$$\Rightarrow P(0 \leq Y \leq y) = \int_0^{y^{1/3}} dx = y^{1/3}.$$

Therefore,

$$F_Y(y) = P(Y \leq y) = \begin{cases} 0, & y \leq 0 \\ y^{1/3}, & 0 \leq y \leq 1 \\ 1, & y \geq 1 \end{cases}$$

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Problem 1. Consider the probability space $(\Omega, \mathcal{F}, P)$. For $A, B \in \mathcal{F}$, write down the definition of what it means for A and B to be independent.

$$P(A \cap B) = P(A) \cdot P(B)$$

Problem 2. Let $A, B \in \mathcal{F}$ for some probability space $(\Omega, \mathcal{F}, P)$. Prove that

$$|P(A) - P(B)| \leq P(A^c \cap B) + P(A \cap B^c).$$

$$\begin{aligned} |P(A) - P(B)| &= |P((A \cap B) \cup (A \cap B^c)) - P((B \cap A) \cup (B \cap A^c))| \\ &= |P(A \cap B) + P(A \cap B^c) - P(B \cap A) - P(B \cap A^c)| \\ &= |P(A \cap B^c) - P(B \cap A^c)| \\ &\leq P(A \cap B^c) + P(B \cap A^c) \end{aligned}$$