

Problem 1. Consider the probability space $(\Omega, \mathcal{F}, P)$. For $A, B \in \mathcal{F}$, write down the definition of $P(A|B)$.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Problem 2. Let X be a random variable on a probability space $(\Omega, \mathcal{F}, P)$ that has the following density

$$f(x) = \begin{cases} 0 & x < 0 \\ x & 0 \leq x \leq 1 \\ 0 & x > 1 \end{cases}$$

Compute the following conditional probability:

$$P\left(X \in \left[\frac{1}{4}, \frac{3}{4}\right] \mid \left[\frac{1}{2}, 1\right]\right).$$

$$\begin{aligned} P\left(X \in \left[\frac{1}{4}, \frac{3}{4}\right] \mid \left[\frac{1}{2}, 1\right]\right) &= \frac{P\left(X \in \left[\frac{1}{4}, \frac{3}{4}\right] \cap \left[\frac{1}{2}, 1\right]\right)}{P\left(\left[\frac{1}{2}, 1\right]\right)} \\ &= \frac{\int_{\frac{1}{2}}^{\frac{3}{4}} x dx}{\int_{\frac{1}{2}}^1 x dx} \\ &= \frac{x^2 \Big|_{\frac{1}{2}}^{\frac{3}{4}}}{x^2 \Big|_{\frac{1}{2}}^1} \\ &= \frac{\frac{9}{16} - \frac{1}{4}}{1 - \frac{1}{4}} \\ &= \frac{\frac{5}{16}}{\frac{3}{4}} \\ &= \frac{5}{12} \end{aligned}$$

Quiz #2

MTH 383/683: Fall 2026

Instructor: John Gemmer

Problem 1. Let X be a positive random variable on a probability space $(\Omega, \mathcal{F}, P)$. Write down Markov's inequality.

$$\text{For } a > 0, P(X > a) \leq \frac{1}{a} \mathbb{E}[X]$$

Problem 2. Let X be a random variable on a probability space $(\Omega, \mathcal{F}, P)$ with moment generating function $\phi(t) = \mathbb{E}[e^{tX}]$. Prove that

$$P(X > a) \leq e^{-at} \phi(t).$$

$$P(X > a) = P(e^{tX} > e^{ta}) \leq \frac{1}{e^{ta}} \mathbb{E}[e^{tX}] = e^{-at} \phi(t).$$