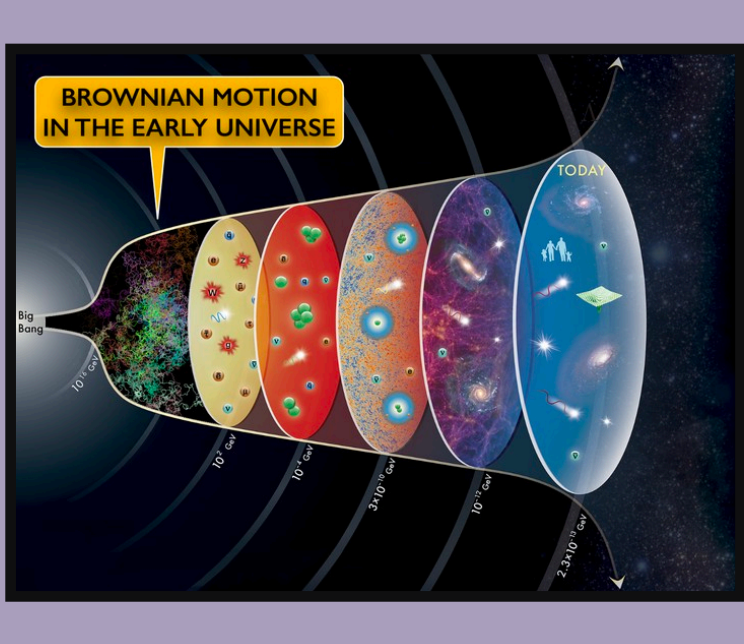
A First Course in Stochastic Calculus



MTH 383/683

Dr. John Gemmer: gemmerj@wfu.edu
Office: Manchester 388

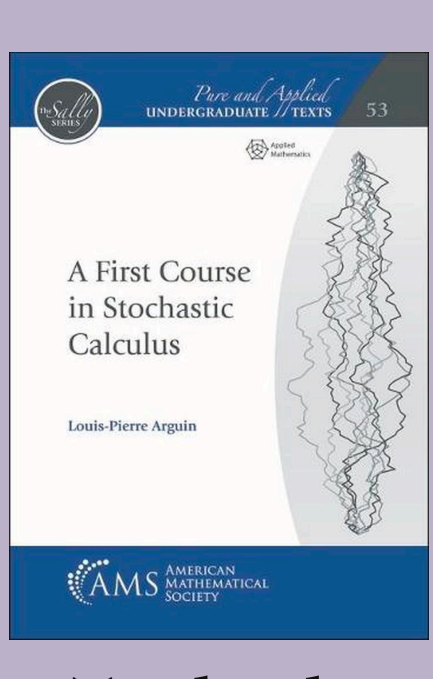
Course Website: http://users.wfu.edu/gemmerj/math383-683F26.html
Canvas: The course syllabus and grades will be posted on Canvas
Office Hours: T 10:00-11:00, W 12:00-2:00, Th 1:00-3:00
Class Meeting Times: MWF 10:00-10:50
Class Location: Kirby-Manchester 241

COURSE DESCRIPTION

The modern theory of stochastic calculus has become an important tool theory of stochastic calculus has become an important tool in modeling the uncertainty in many physical, biological and chemical systems as well as in finance. For example neuronal networks, geophysical flows, climate dynamics, chemical reactions and communication systems are all systems in which random fluctuations can have a significant impact on their evolution. The purpose of this course is for students to learn about Brownian motion and random processes and how to extract quantitative and qualitative information from stochastic models. Moreover, since modern applied mathematics is intrinsically interdisciplinary, a secondary purpose of this course will be to have students apply techniques from the course to specific areas of scientific interest. While students will be exposed to these advanced mathematical topics, these concepts will be made accessible by emphasizing numerical experiments using elementary Python/Matlab coding to build and develop intuition. This approach will be used to elucidate the properties of Gaussian processes, martingales, stochastic differential equations, and diffusions. Along the way, students will learn about applications drawn from physics, biology, chemistry, epidemiology, finance, etc.

REQUIREMENTS

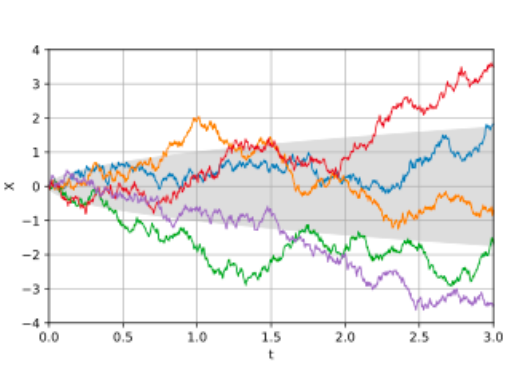
Prerequisites:
Linear Algebra, Differential Equations, Probability



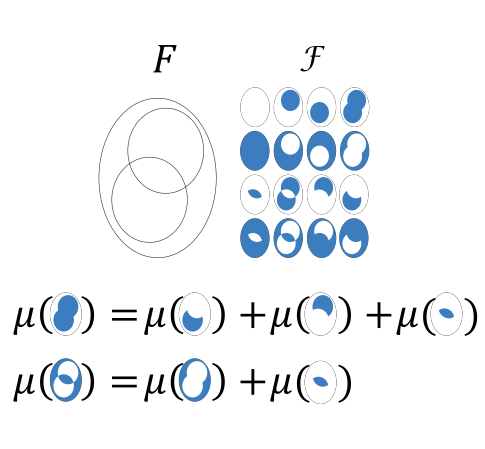
Textbook:
A First Course in Stochastic Calculus

Software:
Matlab, Mathematica, Python: WFU Software Link

OBJECTIVES

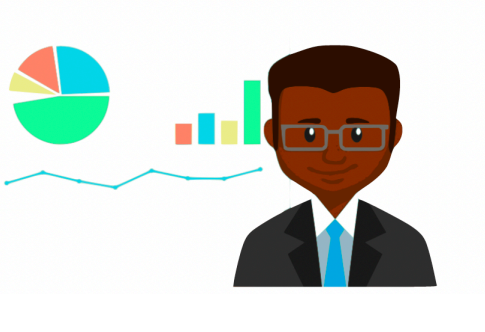


Master Analysis of SDES



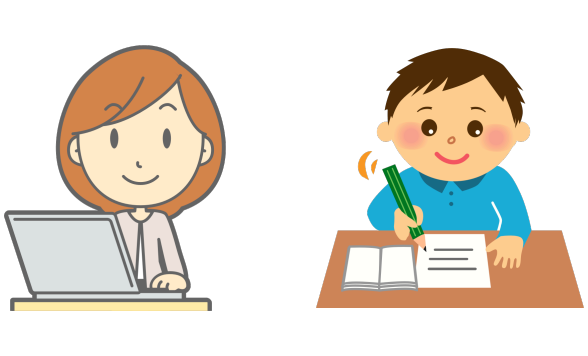
Master Advanced Probability

Develop Programming Skills



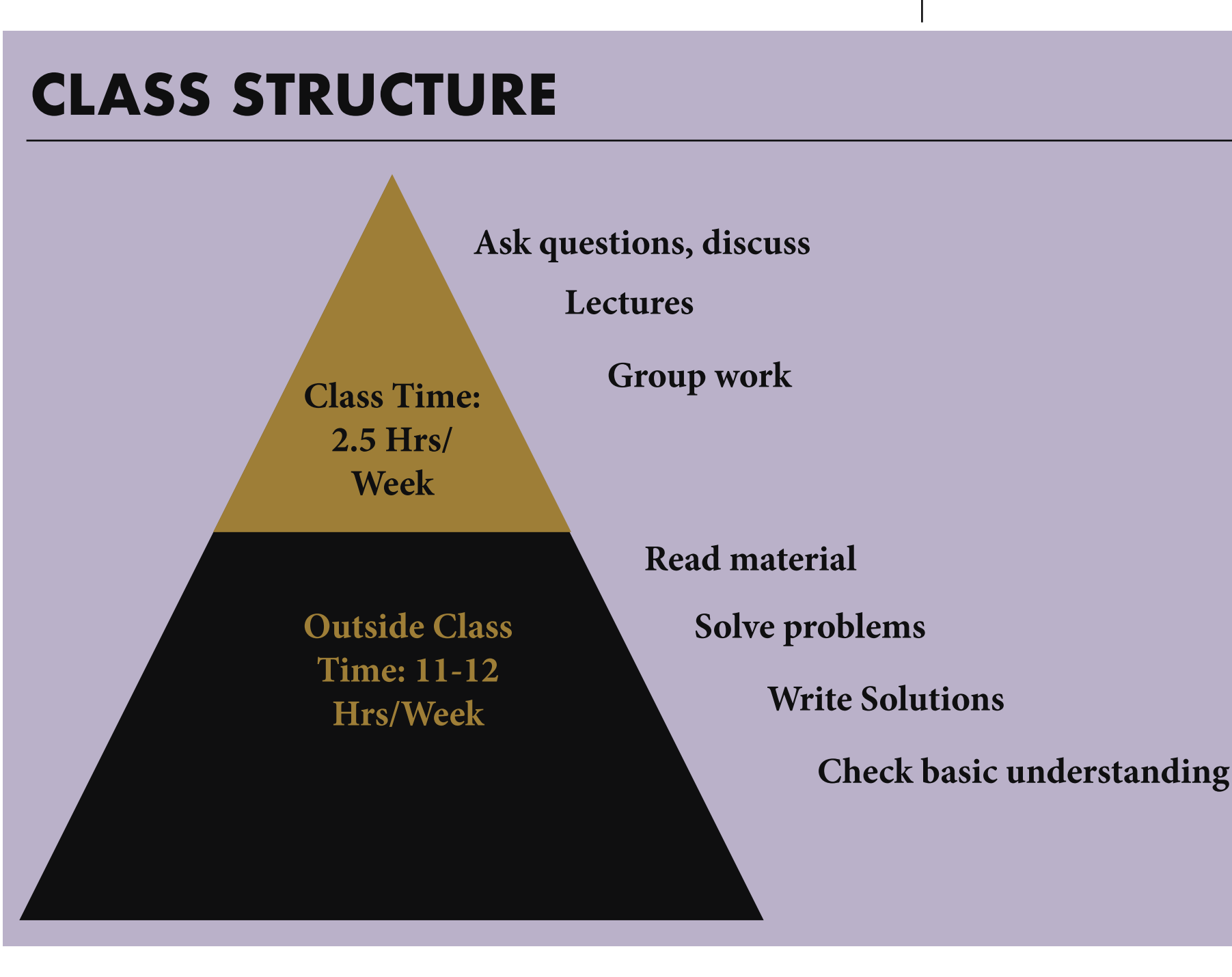
Communicate Effectively

Learning to collaborate



Write Effectively

CLASS STRUCTURE



EVALUATION

We focus on learning and mastery. You are guaranteed the following grades if your final percentage lies within the following ranges:

90-92.9: A-	93-100: A	
80-82.9: B-	83-86.9: B	87-89.9: B+
70-72.9: C-	73-76.9: C	77-79.9: C+
60-62.9: D-	63-66.9: D	67-69.9: D+

Weekly Homework (10%), at least 1 dropped
Open book, collaboration allowed with citation
Homework is due on most Fridays in class

In Class Quizzes Quizzes (10%), at least 1 dropped
5-10 minutes
In class on most Fridays

Computational Assignments (15%)
Structured in class group assignments
Grades based on attendance and completion of assignment

Two summative assessments (30%)
In class, closed notes

Final Exam (35%)
Comprehensive
In class, closed notes

Weekly Homework: Homework will be assigned most weeks on Thursday and will be due Friday in class the following week. Late homework will not be accepted under any circumstances. However, I will drop at least one homework assignment from your grade. While you are allowed to collaborate with your colleagues, homework must consist of solutions that show all steps, be your own work and be written clearly using complete sentences as appropriate (see homework policy). All homework will be submitted in class on paper. I will not accept digital versions of your homework.

Computational Assignments: Throughout the course there will be a several announced and unannounced computational assignments. These will consist of structured coding assignments that will be introduced completed during class time. These assignments will allow students to explore concepts we are studying in class by simulating random processes. The assignments will be completed in Matlab or Python and will be due the following week. These assignments will be graded based on attendance and completion of the assigned tasks.

In Class Quizzes: On most Fridays there will be a short 5-10 minute in-class quizzes. These quizzes will consist of a very short problem that will test your knowledge of the prior lectures and homework. These quizzes are to help both the students and the instructor understand concepts that students may be struggling with. All quizzes will be announced in class. There will be no "pop" quizzes. There are no retakes for missed quizzes, however I will drop the lowest quiz score from your final grade.

Summative Assessments: There will be two in class summative assessments in the course and a cumulative final exam.

Graduate Students: Graduate students will be expected to complete additional homework problems that focus more on the theoretical underpinnings in the course. In many cases, this type of mathematics will develop your expertise in analysis and measure theory. You will also be expected to complete more advanced theoretical problems on the exams.

Late Work Policy: Except in very extreme circumstances, I do not accept late assignments or reschedule exams. If you have a situation in which you cannot make an exam for personal reasons, you must arrange accommodations with the instructor to schedule the exam **before the scheduled exam date.** If you have a legitimate emergency situation, I will make sure that all students in the course will have access to the same exception to this policy.

If you need to miss class due to a university sponsored activity, such as athletics. Please contact the faculty member as soon as possible to reschedule due dates.

COURSE ENVIRONMENT

Names/Pronouns

You **deserve** to be addressed in the manner you prefer. To guarantee that I address you properly, you are welcome to tell me your pronoun(s) and/or preferred name at any time, either in person or via email.

Diversity

We embrace diversity of age, background, beliefs, ethnicity, gender, gender identity, gender expression, national origin, religious affiliation, sexual orientation, and other visible and non-visible categories. **I do not tolerate discrimination.**

Accessibility

I want you to succeed in this course. Wake Forest University provides reasonable accommodations to students with disabilities. If you are in need of an accommodation, then please contact me privately as early in the term as possible. Retroactive accommodations may not be provided. Students requiring accommodations must also consult the Center for Learning, Access, and Student Success(118 Reynolda Hall, 336-758-5929, http://class.wfu.edu). For personal issues, stress, health problems or life circumstances see shs.wfu.edu/. Contact me if you have other special circumstances. **I will find resources for you.**

Title IX

You **deserve** a community free from discrimination, sexual harassment, a hostile environment, sexual assault, domestic violence, dating violence, and stalking. If you experience or know of a Title IX violation, you have many options for support and/or reporting; see titleix.wfu.edu/.

Emergency Fund

You **deserve** a learning environment in which all of your physiological and safety needs are met. If you are experiencing situations in which these needs are not met, e.g. you do not have adequate housing or sufficient food security, the Chaplain's Office has an emergency fund which can provide support: https://chaplain.wfu.edu/care-support/chaplains-emergency-fund/. In situations in which you need immediate assistance there is emergency funding available through the Department of Mathematics and Statistics. If you are in need of emergency help you are encouraged to reach out to a faculty member in the Department of Mathematics and Statistics who will work with the chair of the department to address your needs.

Course Resources

The department has a limited amount of funding for class materials. If you cannot afford class materials, you are encouraged to contact the chair of the department privately as early in the term as possible. Due to the limited amount of funds, students must exhaust all other sources of funding before applying to the department for assistance.

The Honor Code

At Wake Forest, we expect you to behave as honorable citizens of the class, the university, and the world as a whole. When you complete an assignment with your name on it, you are representing that everything you are turning in is your own work. That means that you do not copy from other students, textbooks, or websites. If at any time I become aware of cheating or plagiarism in this course, I will submit the information to the honor council.

TENTATIVE COURSE CALENDAR

Gaussian Processes and Brownian Motion

Week 1:
1. 8/24: Going over the syllabus, Probability Spaces, **Section 1.1**
2. 8/26: Random Variables and Their Distributions, **Section 1.2**
3. 8/28: Introduction to Matlab and Numerical Simulations, **HW #1 Due**

Week 2:
1. 8/31: Random Variables and Their Distributions, Expectation, **Sections 1.2-1.3**
2. 9/2: Expectation, Important Inequalities, **Sections 1.3-1.4**
3. 9/4: **Computational Assignment #1, HW #2 Due**

Week 3:
1. 9/7: **Labor Day (No Classes)**
2. 9/9: Random Vectors, **Section 2.1**
3. 9/11: Gaussian Vectors, **Section 2.2, HW #3, CA#1 Due, Quiz #1**

Week 4:
1. 9/14: Gaussian Processes, **Section 2.3**
2. 9/16: A Geometric Point of View, **Section 2.4**
3. 9/18: **Computational Assignment #2, HW #4 Due, Quiz #2**

Week 5:
1. 9/21: Properties of Brownian Motion I, **Section 3.1**
2. 9/23: Properties of Brownian Motion II, **Section 3.2**
3. 9/25: **Summative Assessment #1, CA #2 Due**

Expectation and Martingales

Week 6:
1. 9/28: Conditional Expectation I, **Section 4.1**
2. 9/30: Conditional Expectation II, **Section 4.2**
3. 10/2: **Computational Assignment #3, HW #4 Due, Quiz #3**

Week 7:
1. 10/5: Conditional Expectation III, **Section 4.2**
2. 10/7: Martingales I**, Section 4.3**
3. 10/9: Martingales II, **Section 4.3, HW #5 Due, CA #3 Due, Quiz #4**

Week 8:
1. 10/12: **Fall Break (No Classes)**
2. 10/14: Martingales III **Sections 4.3-4.4**
3. 10/16: **Computational Assignment #4, HW #5 Due, Quiz #5**

Week 9:
1. 10/19: Martingales IV, **Section 4.4,**
2. 10/21: Reflection Principle for Brownian Motion, **Section 4.5**
3. 10/23: Martingale Transform I, **Section 5.1, HW #6, CA #4 Due, Quiz #6**

Week 10:
1. 10/26: Martingale Transform II, **Sections 5.2**
2. 10/28: **Catch Up Day**
3. 10/30: **Summative Assessment #2**

Ito Calculus and Stochastic Differential Equations

Week 11:
1. 11/2: Ito Integral I, **Section 5.3**
2. 11/4: Ito Integral II, **Section 5.3**
3. 11/6: **Computational Assignment #5, HW #7 Due, Quiz #7**

Week 12:
1. 11/9: Ito Formula I, **Section 5.4**
2. 11/11: Ito Formula II, **Section 5.4**
3. 11/13: Gambler's Ruin for Brownian motion with Drift**, HW #8, CA #5 Due, Quiz #8**

Week 13:
1. 11/16: Stochastic Differential Equations I, **Section 7.1**
2. 11/18: Stochastic Differential Equations II, **Section 7.2**
3. 11/20: Numerical Simulation of SDEs, **Section 7.4, HW #9 Due, Quiz #9**

Week 14:
1. 11/23: **Computational Assignment #6**
2. 11/25: **Thanksgiving break**
3. 11/27: **Thanksgiving break**

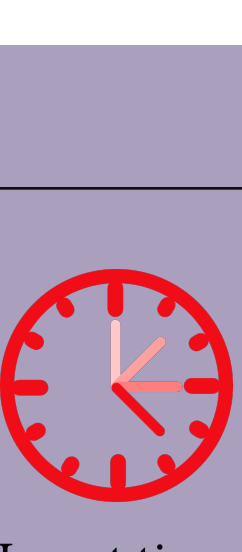
Week 15:
1. 11/30: The Markov Property for Diffusions, **Section 8.1**
2. 12/2: Kolmogorov's Equations I, **Section 8.3**
3. 12/4: Kolmogorov's Equation II, **Section 8.3, HW #10, CA #6 Due, Quiz #10**

Week 16:
1. 12/7: **Catch Up Day**

SUCCESS

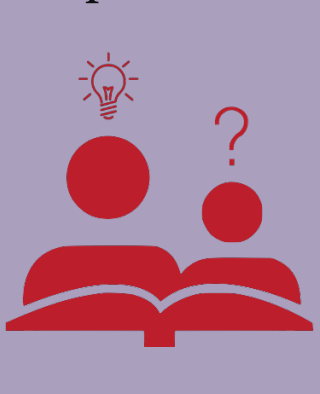
Attend class

Participate constantly



Invest time

Concentrate on concepts in addition to calculations



Seek help when needed

Eliminate Virtual Distractions