Quiz 1

Analysis

January 25, 2018

1. Let a_n be a sequence in $\mathbb{R}$ satisfying for all $n \in \mathbb{N}$

$$|a_{n+1} - a_n| < \frac{1}{n^{3/2}}.$$

Is a_n a Cauchy sequence? Prove or give a counterexample.

Yes, Let $m, n \in \mathbb{N}$ and without loss of generality assume $m \geq n$. Therefore,

$$|a_m - a_n| \leq |a_m - a_{m-1}| + |a_{m-1} - a_{m-2}| + \dots + |a_{n+1} - a_n|$$
$$\leq \frac{1}{m^{3/2}} + \dots + \frac{1}{n^{3/2}}$$
$$= \sum_{k=n}^{m} \frac{1}{k^{3/2}}$$
$$\leq \sum_{k=n}^{\infty} \frac{1}{k^{3/2}}$$

Since $\sum_{k=n}^{\infty} \frac{1}{k^{3/2}}$ converges it follows that $\lim_{n \to \infty} \sum_{k=n}^{\infty} \frac{1}{k^{3/2}} = 0$. ■