Quiz 1

Analysis

January 25, 2018

1. Let \( a_n \) be a sequence in \( \mathbb{R} \) satisfying for all \( n \in \mathbb{N} \)

\[
|a_{n+1} - a_n| < \frac{1}{n^{3/2}}.
\]

Is \( a_n \) a Cauchy sequence? Prove or give a counterexample.

Yes, let \( m, n \in \mathbb{N} \) and without loss of generality assume \( m > n \). Therefore,

\[
|a_m - a_n| \leq |a_m - a_{m-1}| + |a_{m-1} - a_{m-2}| + \ldots + |a_{n+1} - a_n|
\]

\[
= \frac{1}{m^{3/2}} + \frac{1}{(m-1)^{3/2}} + \frac{1}{(m-2)^{3/2}} + \ldots + \frac{1}{n^{3/2}}
\]

\[
= \sum_{k=n}^{m-1} \frac{1}{k^{3/2}}
\]

Since \( \sum_{k=n}^{\infty} \frac{1}{k^{3/2}} \) converges it follows that \( \lim_{n \to \infty} \sum_{k=n}^{\infty} \frac{1}{k^{3/2}} = 0 \).