

Quiz 10

Analysis

April 20, 2018

1. Define $K : C([0, 1], \|\cdot\|_{L^2}) \mapsto C([0, 1], \|\cdot\|_{L^2})$ by

$$K(f(x)) = \int_0^1 k(x, y) f(y) dy,$$

where $k : [0, 1] \times [0, 1] \mapsto \mathbb{R}$ is continuous and $k(x, y) \geq 0$. Prove that K is bounded.

Let $M = \max_{x, y} k(x, y)$. Then,

$$\begin{aligned} \int_0^1 K(f(x))^2 dx &= \int_0^1 \left(\int_0^1 k(x, y) f(y) dy \right)^2 dx \\ &\leq \int_0^1 \left(\int_0^1 M \cdot |f(y)| dy \right)^2 dx \\ &\leq M^2 \int_0^1 \int_0^1 f(y)^2 dy dx \\ &= M^2 \cdot \|f\|_{L^2}^2 \end{aligned}$$

$\Rightarrow K$ is bounded.