

Quiz 2

Analysis

February 10, 2018

1. Prove that for every bounded sequence $x_n \in \mathbb{R}$

$$\liminf_{n \rightarrow \infty} x_n \leq \limsup_{n \rightarrow \infty} x_n.$$

Let

$$y_n = \inf \{x_k : k \geq n\},$$

$$z_n = \sup \{x_k : k \geq n\}$$

Therefore,

$$y_n \leq z_n$$

$$\Rightarrow \liminf_{n \rightarrow \infty} x_n = \lim_{n \rightarrow \infty} y_n \leq \lim_{n \rightarrow \infty} z_n = \limsup_{n \rightarrow \infty} x_n.$$

2. Give an example of a bounded sequence $x_n \in \mathbb{R}$ for which the above inequality is strict.

$$x_n = (-1)^n.$$