Quiz 2

Analysis

February 10, 2018

1. Prove that for every bounded sequence \( x_n \in \mathbb{R} \)
\[
\liminf_{n \to \infty} x_n \leq \limsup_{n \to \infty} x_n.
\]

Let
\[
\gamma_n = \inf \{ x_k : k \geq n \},
\]
\[
\zeta_n = \sup \{ x_k : k \geq n \}
\]

Therefore,
\[
\gamma_n \leq \zeta_n
\]
\[
\implies \liminf_{n \to \infty} x_n = \lim_{n \to \infty} \gamma_n \leq \lim_{n \to \infty} \zeta_n = \limsup_{n \to \infty} x_n.
\]

2. Give an example of a bounded sequence \( x_n \in \mathbb{R} \) for which the above inequality is strict.
\[
x_n = (-1)^n.
\]