1 Problems for everyone

1. pg. 149-151, #4.2.3, #4.2.4, #4.2.7, #4.2.8

2. Consider the following initial-boundary value problem:
   \[ u_t = u_{xx}, \quad x \in [0, 2\pi], \]
   \[ u(0, t) = 1, \ u(2\pi, t) = 2 \]
   \[ u(x, 0) = x. \]
   (a) Calculate the steady state solution for this initial-boundary value problem.
   (b) Solve this initial boundary value problem.

3. Consider the following initial-boundary value problem:
   \[ u_t = u_{xx} - u, \quad x \in [0, 1], \]
   \[ u(0, t) = u(1, t) = 0, \]
   \[ u(x, 0) = x(1-x). \]
   (a) Using separation of variables solve this initial-boundary value problem.
   (b) Using your solution, calculate \( \lim_{t \to \infty} u(x, t). \)
   (c) Are there any steady state solutions to this equation? If so, what are they?

4. Consider the following initial-boundary value problem:
   \[ u_{tt} = u_{xx}, \quad x \in [0, \pi], \]
   \[ u_x(0, t) = u_x(\pi, t) = 0, \]
   \[ u(x, 0) = \cos^2(x), \]
   \[ u_t(x, 0) = \cos(3x). \]
   (a) Solve this initial-boundary value problem.
   (b) Using software such as Matlab, Mathematica, etc sketch the solution for \( t = 0, \ t = \pi/2, \)
       \( t = \pi, \ t = 3\pi/2, \) and \( t = 2\pi. \)
   (c) Sketch a contour plot of your solution as a function of \( x \) and \( t. \) (If you want to, you can
       use software to do this.)
   (d) Describe qualitatively the behavior of the solution.
2 Graduate Problems

1. pg. 149-151, #4.2.10, #4.2.11

2. Consider the following initial-boundary value problem:

\[ u_{tt} + u_t = u_{xx}, \quad x \in [0, \pi], \]
\[ u(0, t) = u(\pi, t) = 0, \]
\[ u(x, 0) = \sin^2(x), \]
\[ u_t(x, 0) = 0. \]

(a) Solve this initial-boundary value problem.

(b) Using software such as Matlab, Mathematica, etc sketch the solution for \( t = 0, \ t = \pi/2, \ \ t = \pi, \ t = 3\pi/2, \) and \( t = 2\pi \) using 20 terms in the Fourier series.

(c) Sketch a contour plot of your solution as a function of \( x \) and \( t. \) (If you want to, you can use software to do this.)

(d) Describe qualitatively the behavior of the solution.