MTH 352/652
Quiz #8

Special Directions: This is a small group quiz with the teams assigned below. You are only allowed to work with your teammates. Any intentional communication with other teams will result in a zero for your entire team. Each member of the team will receive the same grade based on the lowest score.

Team #1	**Team #2**	**Team #3**	**Team #4**
Emily	Lyle	Sarah	Ethan
Jasmine	Yiyi	Shelby	Mandy
Nick	Cordell	Jiachen	Colin
Miguel	Cole	Steven	Clay

1. Suppose $\Omega = \{(x, y) \in \mathbb{R}^2 : 1 \leq x^2 + y^2 \leq 4\}$. Solve the following boundary value problem on Ω. **Hint:** You can just use the general form of the solution to Laplace's equation in polar coordinates.

$$\Delta u = 0$$
$$u|_{x^2+y^2=4} = 4 + 3\sin(2\theta) + \cos(2\theta)$$
$$u|_{x^2+y^2=1} = 0$$

$u(r,\theta) = a + b\ln(r) + (cr^2 + dr^{-2})\sin(2\theta) + (fr^2 + gr^{-2})\cos(2\theta)$

$u(1,\theta) = 0 = a + (c+d)\sin(2\theta) + (f+g)\cos(2\theta)$

$\Rightarrow a = 0,\ c = -d,\ f = -g$

$u(2,\theta) = 4 + 3\sin(2\theta) + \cos(2\theta)$

$= b\ln(2) + (c\cdot 2^2 - c\cdot 2^{-2})\sin(2\theta) + (f\cdot 2^2 - f\cdot 2^{-2})\cos(2\theta)$

$= b\ln(2) + \frac{15}{4}c\sin(2\theta) + \frac{15}{4}f\cos(2\theta)$

$\Rightarrow b = \frac{4}{\ln(2)},\ c = 4/5,\ f = 4/15$

$u(r,\theta) = \frac{4}{\ln(2)}\ln(r) + \frac{4}{5}(r^2 - r^{-2})\sin(2\theta) + \frac{4}{15}(r^2 - r^{-2})\cos(2\theta)$