

key

MTH 225

Quiz #1

1. Find all solutions to the following system of linear equations over  $\mathbb{Z}_3$ :

$$2x + 2y + z = 2,$$

$$2x + y + 2z = 1.$$

$$\begin{aligned} \left[ \begin{array}{ccc|c} 2 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \end{array} \right] \times 2 &\rightarrow \left[ \begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 2 & 1 & 2 & 1 \end{array} \right] + R_1 &\rightarrow \left[ \begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 2 & 1 & 2 \end{array} \right] + R_2 \\ &\rightarrow \left[ \begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 2 & 1 & 2 \end{array} \right] \times 2 &\rightarrow \left[ \begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 1 \end{array} \right] \end{aligned}$$

Therefore,

$$\begin{aligned} x=0 &\Rightarrow x=0 \\ y+2z=1 &\Rightarrow y=1+z \end{aligned}$$

Consequently, the solutions are

$$\begin{array}{l} x=0 \quad x=0 \quad x=0 \\ y=1 \quad y=2 \quad y=0 \\ z=0 \quad z=1 \quad z=2 \end{array}$$