key

MTH 225
Quiz #2

1. Let $W \subset C^0(\mathbb{R})$ be defined by

$$W = \left\{ f \in C^0(\mathbb{R}) : \int_0^1 f(x)dx = 0 \right\}.$$

Prove that W is a vector space by proving that it is a subspace of $C^0(\mathbb{R})$.

Let $f, g \in W$ and $\lambda \in \mathbb{R}$. Therefore,
$\int_0^1 f(x)dx = 0$ and $\int_0^1 g(x)dx = 0$.
Consequently, if $h(x) = f(x) + \lambda g(x)$ then

$$\begin{aligned}\int_0^1 h(x)dx &= \int_0^1 (f(x) + \lambda g(x))dx \\ &= \int_0^1 f(x)dx + \lambda \int_0^1 g(x)dx \\ &= 0 + \lambda \cdot 0 \\ &= 0.\end{aligned}$$

Therefore, $h \in W$ proving that W is a subspace of $C^0(\mathbb{R})$.