

MTH 225

Quiz #3

1. Let $T : M_{2 \times 2}(\mathbb{R}) \mapsto P_2(\mathbb{R})$ be the linear transformation defined by

$$T \left(\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \right) = a_{11} + a_{12} + (a_{12} + a_{21})x + (a_{21} + a_{22})x^2.$$

and let

$$\mathcal{A} = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\} \text{ and } \mathcal{B} = \{1, x, x^2\}$$

be the standard bases for $M_{2 \times 2}(\mathbb{R})$ and $P_2(\mathbb{R})$ respectively. Find the matrix representation of T with respect to \mathcal{A} and \mathcal{B} , i.e., find $[T[\mathcal{A}, \mathcal{B}]$.

$$T \left(\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \right) = 1$$

$$T \left(\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \right) = 1 + x$$

$$T \left(\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \right) = x + x^2$$

$$T \left(\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right) = x^2$$

$$\Rightarrow [T[\mathcal{A}, \mathcal{B}]] = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$