

MTH 225

Quiz #5

1. Suppose $\langle \cdot, \cdot \rangle$ is the standard complex inner product on \mathbb{C}^n and $\mathbf{u}, \mathbf{v} \in \mathbb{C}^n$ satisfy

$$\|\mathbf{u}\| = 1, \|\mathbf{v}\| = 2, \text{ and } \langle \mathbf{u}, \mathbf{v} \rangle = 1 + i.$$

Use all of this information to compute and simplify

$$\langle i\mathbf{u} + \mathbf{v}, \mathbf{u} - \mathbf{v} \rangle.$$

$$\begin{aligned} \langle i\mathbf{u} + \mathbf{v}, \mathbf{u} - \mathbf{v} \rangle &= \langle i\mathbf{u}, \mathbf{u} \rangle + \langle i\mathbf{u}, -\mathbf{v} \rangle + \langle \mathbf{v}, \mathbf{u} \rangle + \langle \mathbf{v}, -\mathbf{v} \rangle \\ &= -i\langle \mathbf{u}, \mathbf{u} \rangle + i\langle \mathbf{u}, \mathbf{v} \rangle + \overline{\langle \mathbf{v}, \mathbf{u} \rangle} - \langle \mathbf{v}, \mathbf{v} \rangle \\ &= -i\|\mathbf{u}\|^2 + i\langle \mathbf{u}, \mathbf{v} \rangle + \overline{\langle \mathbf{u}, \mathbf{v} \rangle} - \|\mathbf{v}\|^2 \\ &= -i + i(1+i) + (1-i) - 4 \\ &= -i + i - 1 + 1 - i - 4 \\ &= -i - 4 \\ &= -4 - i \end{aligned}$$