MTH 352/652
Computational Assignment #2

First Due Date: April 11, 2025

Ground Rules:

1. You will submit your assignments via e-mail. Each individual problem should be a Mathematica Notebook. Format the assignments by naming your notebooks LastnameProblemnumber.nb. For example, for the second problem I would save my Mathematica Notebook as Gemmer2.nb.

2. You cannot use ChatGPT or copy code from the internet or your classmates. However, you are more than welcome to work with your classmates and you can use the internet to get ideas of how to write code. For example, searching for something like "how to define a piecewise function in Mathematica" is totally fine. Searching for something like "Give me code for solving the heat equation with a step function" is not good.

3. You can hand in the assignment as many times as you would like as long as you hand in your assignment weekly by Friday at 11:00 AM. If you miss any week you will earn the grade of the last submitted assignment. This is to prevent students from putting off the assignment until the end of the semester.

1 Problem #1

Modify the "Fourier Series Example" Mathematica Notebook to plot the first five partial sums of the the Fourier series expansion of the function $f(x) = x(\pi - x)(x - \pi/4)$ on the interval $[0, \pi]$. Your code should

1. Plot $f(x)$ on the same plot as the first five partial sums that approximate $f(x)$.

2. Create an animation that shows the convergence of the Fourier series using the first five partial sums.

3. Create a "spectral diagram" that illustrates the different Fourier modes.

In each of your plots you must use appropriate bounds to adequately illustrate the behavior of the plot.

2 Problem #2

Modify the code "Heat Equation on a Bounded Interval" Mathematica Notebook on the course website to solve the heat equation

$$\begin{aligned} u_t &= u_{xx} \\ u(0,t) &= 0 \\ u(\pi,t) &= 0 \\ u(x,0) &= x(x-\pi) \end{aligned}$$

Your code should:

1. At times $t = 0, .1, .2, .5, 1, 2$, plot the solution on the interval $[0, \pi]$ using a partial sum with 10 terms.
2. Plot a density or contour plot on the domain $x \in [0, \pi]$, $t \in [0, 2]$ using a partial sum with 10 terms.
3. Use the animate command to plot a movie of the evolution of the solution on the spatial domain $[0, \pi]$ for times t ranging from 0 to 2 using partial sums with 10 terms.

3 Problem #3

Modify the code "Heat Equation on a Bounded Interval" Mathematica Notebook on the course website to solve the heat equation

$$\begin{aligned} u_t &= u_{xx} \\ u_x(0,t) &= 0 \\ u_x(\pi,t) &= 0 \\ u(x,0) &= x^2(x-\pi)^2 \end{aligned}$$

Your code should:

1. At times $t = 0, .1, .2, .5, 1, 2$, plot the solution on the interval $[0, \pi]$ using a partial sum with 10 terms.
2. Plot a density or contour plot on the domain $x \in [0, \pi]$, $t \in [0, 2]$ using a partial sum with 10 terms.
3. Use the animate command to plot a movie of the evolution of the solution on the spatial domain $[0, \pi]$ for times t ranging from 0 to 2 using partial sums with 10 terms.

4 Problem #4

Modify the code "Steady State Solutions to Heat Equation" Mathematica Notebook on the course website to solve the heat equation

$$\begin{aligned} u_t &= u_{xx} \\ u(0,t) &= 0 \\ u(1,t) &= 1 \\ u(x,0) &= x(x-1)+x \end{aligned}$$

Your code should:

1. At times $t = 0, .1, .2, .5, 1, 2$, plot the solution on the interval $[0,1]$ using a partial sum with 10 terms.
2. Plot a density or contour plot on the domain $x \in [0,1]$, $t \in [0,2]$ using a partial sum with 10 terms.
3. Use the animate command to plot a movie of the evolution of the solution on the spatial domain $[0,1]$ for times t ranging from 0 to 2 using partial sums with 10 terms.