MTH 352
Quiz #4

1. Consider the following partial differential equation on $\mathbb{R}$:

$$u_t - \sin(t)u_x = 0.$$

Show that by changing variables to $X = x - \cos(t)$ and $\tau = t$ that this equation be expressed in the form

$$u_\tau = 0.$$

$$\frac{\partial}{\partial x} = \frac{\partial X}{\partial x}\frac{\partial}{\partial X} + \frac{\partial \tau}{\partial x}\frac{\partial}{\partial \tau}$$

$$= \frac{\partial}{\partial X}$$

$$\frac{\partial}{\partial t} = \frac{\partial X}{\partial t}\frac{\partial}{\partial X} + \frac{\partial \tau}{\partial t}\frac{\partial}{\partial \tau}$$

$$= \sin(t)\frac{\partial}{\partial X} + \frac{\partial}{\partial \tau}.$$

Therefore,

$$0 = u_t - \sin(t)u_x = \sin(t)u_X + u_\tau - \sin(t)u_X$$

$$\Rightarrow 0 = u_\tau.$$